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# A variable flow modelling approach to military end strength planning

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# NAVAL POSTGRADUATE SCHOOL

**MONTEREY, CALIFORNIA**

# THESIS

**A VARIABLE FLOW MODELLING APPROACH TO MILITARY END STRENGTH PLANNING**

by

Benjamin K. Grossi

December 2016

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I compare forecasts using current and optimized parameters against each other, and the results show that optimizing transition probabilities is the most efficient way of meeting manpower targets—while maintaining the current hiring policy—for the Marine Technician category.

I also conduct a risk analysis by simulating the effect of changes in the transition rate on the differential between the forecast and desired end strengths. Again, the transition probability optimization model performs better than the status quo situation.

Recommendations are made for future research to improve the implementation of optimized transition probabilities and also for ways of limiting the attrition rate, which is the only variable not under the control of the Royal Australian Navy.

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**A VARIABLE FLOW MODELLING APPROACH TO MILITARY END STRENGTH PLANNING**

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Submitted in partial fulfillment of the
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## TABLE OF CONTENTS


I. INTRODUCTION ..... 1
- A. VARIABLE FLOW MODEL ..... 2
- B. RESEARCH QUESTIONS ..... 2

II. LITERATURE REVIEW ..... 5
- A. QUANTITATIVE WORKFORCE PLANNING MODELS ..... 5
  - 1. System Dynamics ..... 5
  - 2. Computer Simulation Models ..... 6
  - 3. Optimization Models ..... 6
  - 4. Markov Chain Models ..... 7
- B. MARKOV MODELLING APPROACHES ..... 8

III. DATA AND METHODOLOGY ..... 11
- A. DATA ..... 11
- B. METHODOLOGY ..... 12
  - 1. Data Selection ..... 12
  - 2. Annual Transition Probabilities ..... 12
  - 3. Estimation of Transition and Distribution Probabilities ..... 14
  - 4. Matrix Calculations ..... 15
- C. SCENARIOS ..... 16
  - 1. Base Case ..... 18
  - 2. Accession Optimization Scenario ..... 18
  - 3. Transition Probability Optimization Scenarios ..... 19
- D. SUMMARY ..... 24

IV. RESULTS AND RISK ANALYSIS ..... 25
- A. BASE CASE RESULTS ..... 25
- B. ACCESSION OPTIMIZATION RESULTS ..... 27
- C. TRANSITION PROBABILITY OPTIMIZATION SCENARIOS ..... 30
  - 1. Scenario 3A ..... 30
  - 2. Scenario 3B ..... 32
  - 3. Scenario 3C ..... 35
  - 4. Scenario 3D ..... 37
- D. RISK ANALYSIS ..... 39
- E. SUMMARY ..... 43

V. LIMITATIONS, RECOMMENDATIONS AND CONCLUSION .................45
A. LIMITATIONS .........................................................................45
B. RECOMMENDATIONS...........................................................46
C. CONCLUSION ........................................................................46

APPENDIX. HISTORICAL TRANSITION MATRICES ..........................................49

LIST OF REFERENCES ...................................................................................51

INITIAL DISTRIBUTION LIST ........................................................................53

# LIST OF FIGURES


Figure 1. MT End Strength FY 14–15 (Adapted from DWMFA, 2016). ................13

Figure 2. MT transition probabilities FY 14–15 .....................................................13

Figure 3. Estimated Transition Matrix ....................................................................14

Figure 4. Example Objective Function Calculation. ................................................17

Figure 5. Example of a Fundamental Matrix. .........................................................20

Figure 6. Minimum Constraint Matrix for Transition Probability Optimization. .....20

Figure 7. Maximum Constraint Matrix for Transition Probability Optimization. ...........................................................................................21

Figure 8. Objective Function Weighting for Forecast Difference from Desired End Strength.........................................................................................23

Figure 9. Example of Weighting Objective Function by Rank. ...............................24

Figure 10. Scenario 1—Base Case Hires and Distribution of Personnel ...................26

Figure 11. Scenario 1—Base Case End Strength Forecast. .......................................26

Figure 12. Scenario 2— Hires and Distribution. .......................................................28

Figure 13. Scenario 2—End Strength Forecast. .........................................................29

Figure 14. Scenario 3A—Optimized Transition Matrix. ...........................................30

Figure 15. Scenario 3A—Fundamental Matrix Corresponding to Optimized Transition Matrix in Figure 14. .................................................................31

Figure 16. Scenario 3A—Forecast End Strength Using Transition Matrix in Figure 14. ..........................................................................................31

Figure 17. Scenario 3B—Optimized Transition Matrix. ............................................33

Figure 18. Scenario 3B—Fundamental Matrix Corresponding to Optimized Transition Matrix in Figure 17. .................................................................33

Figure 19. Scenario 3B—Forecast End Strength Using Transition Matrix in Figure 17. ..........................................................................................34

Figure 20. Scenario 3C—Optimized Transition Matrix. ............................................35

Figure 21. Scenario 3C—Fundamental Matrix Corresponding to Optimized Transition Matrix in Figure 20 ....................................................................35

Figure 22. Scenario 3C—Forecast End Strength Using Transition Matrix in Figure 20. ...........................................................................................36

Figure 23. Scenario 3C—Optimized Transition Matrix. ............................................37

Figure 24. Scenario 3C—Fundamental Matrix Corresponding to Optimized Transition Matrix in Figure 23 ....................................................................37

Figure 25. Scenario 3C—Forecast End Strength Using Transition Matrix in Figure 23. ...........................................................................................38

Figure 26. Base Case Transition Probabilities Risk Analysis Over One Year...........40

Figure 27. Optimized Transition Probabilities Risk Analysis Over One Year ...........41

Figure 28. Base Case Risk Analysis Over Seven Years.............................................41

Figure 29. Optimized Transition Probabilities Over Seven Years ..............................42

# LIST OF TABLES


Table 1. Estimated Distribution Probabilities (Adapted from DWMFA, 2016)......15

Table 2. Table of Scenarios..........................................................................17

Table 3. Scenario 1 Steady State Analysis Results..................................................27

Table 4. Differences Between Scenario 1 and 2......................................................29

Table 5. Scenario 3A Steady State Comparison......................................................32

Table 6. Steady State Comparison of Scenarios 3A and 3B....................................34

Table 7. Steady State Comparison of Scenarios 3B and 3C....................................36

Table 8. Steady State Comparison of Scenarios 3C and 3D....................................39

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# LIST OF ACRONYMS AND ABBREVIATIONS

| | |
|---|---|
| ANAO | Australian National Audit Office |
| DWMFA | Directorate of Workforce Modelling, Forecasting and Analysis |
| FY | financial year |
| LP | linear programming |
| MAPE | mean average percentage error |
| MLRPS | Manpower Long-Range Planning System |
| MT | marine technician |
| OR | operations research |
| RAN | Royal Australian Navy |
| SD | system dynamics |
| SMCR | Selected Marine Corps Reserve |

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# I. INTRODUCTION

The purpose of this thesis is to develop a model to assist military manpower planners in meeting prescribed end strength requirements.

End strength is a general term which is used to describe the total number of personnel in a force, or a particular sub-element of the force, at any given time. For the purposes of this thesis, the end strength will refer to each rank within a particular sub-element of a force.

The ability to forecast end strength of a rank is determined by three factors:

1. the beginning strength, which is the end strength at the completion of the previous time interval;
2. the number of personnel entering the rank, which is either by hiring or promotion; and
3. the number of personnel exiting the rank, which is either by attrition or promotion.

The ability to meet end strength is extremely important as reaching the target numbers will provide a military force the greatest probability of delivering, in the most efficient manner, the capability required by its government. Problems arise when, over a period of time, factors either out of the control of workforce planners, or unknowingly within their control, result in manpower shortages. If numbers fall short, extra pressure and workload will be placed on existing personnel to deliver the same capability. This could lead to a further increase in end strength deficit due to increased attrition. If numbers are above those required, then some personnel are effectively being paid with no return benefit to the force. These are both situations which workforce planners are required to address and minimize the effects of, otherwise it may result in a situation where a force is unable to deliver a required capability. This thesis will provide tools to assist planners in their efforts.

## A. VARIABLE FLOW MODEL

The term “variable flow” in the model I have developed refers to its ability to minimize forecast end strength deficits by varying either the flow of personnel into the system, or the flow rate of personnel through the system once they are already in. The flow of personnel into the system is determined by a combination of new hires, also called accessions, and the distribution of these accessions into different ranks in the system. The flow rate of personnel through the system is determined by the amount of time they spend in a rank, and is affected by both promotion and attrition rates.

Identification of the optimal flow, or transition, rate through the system is essential for organizations constrained by the number of personnel they can hire each year. To demonstrate the effect of a sub-optimal transition rate, I will use the Marine Technician (MT) branch of the Royal Australian Navy (RAN) as a case study.

The MT branch has a history of shortages across all ranks. In fact, a report from the Australian National Audit Office (ANAO) stated “The marine technician employment category is classified by Defence as ‘critical’, and has been classified as either ‘critical’ or ‘perilous’ since 1999. The category is not expected to recover in the next decade” (2014). The variable flow model will not be able to solve the MT manpower issues on its own, but it will provide optimal transition rates and end strength targets that the RAN should aim to achieve with its current hiring policy, and provide forecasts for any given increase in hiring rate that the RAN sees fit to employ.

## B. RESEARCH QUESTIONS

The variable flow model combines a Markov modelling approach with linear programming (LP) optimization to give its forecasts. In order to justify the approach I’ve taken, and prove the relevance and validity of the model, the remainder of this thesis will revolve around answering the following research questions:

1. Can a Markov Modelling approach aid in solving end strength problems in the RAN?

    i) Can a Markov Model be built to predict MT end strength?
    ii) Can a linear program be developed to optimize accessions and transition rates in the MT ranks?

iii) What are some of the shortcomings of the Markov model approach as applied to the MT branch?

2. Can Simulation be used to estimate the risk of falling below or above end strength targets?

i) Can Simulation be used to estimate end strength target risks in the MT ranks?

ii) What data are required to use simulation to estimate such risks?

To answer these questions, I have conducted a review of relevant literature to compare the modelling decisions I have made with other research in this area. I discuss the methodology followed in building the model, and outline scenarios in which I have adjusted parameters of the model, and conducted optimizations that are aimed at minimizing the difference between forecast and desired end strength. I conduct a risk analysis simulation to determine the validity and potential accuracy of the model. Using the results from the model, I have provided recommendations on hiring strategies and policy adjustments that best enable future end strength requirements to be met. I also discuss limitations of the model and propose ideas for future research.

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# II. LITERATURE REVIEW

The purpose of this literature review is to identify previous studies where research has been conducted into end strength planning and discuss the relevance of these studies to the methodology employed in this thesis. The research reviewed will also include discussion on alternative modelling methods, and provide justification for the options chosen in this thesis.

## A. QUANTITATIVE WORKFORCE PLANNING MODELS

Wang (2005) conducted a study into operations research (OR) techniques employed in the manpower planning field. He identified four major model categories that may be applied to solving problems similar to the MT end strength problem. These are:

- System dynamics
- Optimization models
- Computer simulation models
- Markov chain models

I will provide a brief description of each of these modelling techniques and assess their relevance to the methods employed in this thesis.

### 1. System Dynamics

According to Wang, a System Dynamics (SD) model is ideal for strategic analysis as it encompasses all the behaviours of a system and how the behaviours are influenced by policy (2005). The model relies on an accurate conceptualization of the workforce structures and associated interactions of feedback loops. Richardson states that the feedback concept is at the heart of the system dynamics approach. A feedback loop exists when information resulting from some action travels through a system and eventually returns in some form to its point of origin, potentially influencing future action (1996).

The downside of SD modelling is the difficulty in conceptualizing a real situation into a causal loop. Wang claims that there is a lack of formal procedure in the art of conceptualization, which makes it difficult for beginners in the field (2005). The data provided for this thesis is not sufficient for feedback analysis and the conceptualization of the MT workforce structure is too extensive for the scope of this thesis, therefore an SD model approach is not employed in this model.

### 2. Computer Simulation Models

Wang defines simulation as a technique that mimics real-world systems, where the system elements have some kind of mathematical or statistical relationship which is too complex to model using analytical techniques, such as Markov chains (2005). For this thesis, the mathematical relationships are not overly complex, so I have chosen not to forecast end strength using computer simulations.

Wang goes on to describe simulation processes as most suitable for answering “what if…” type scenarios by outputting system performance results based on differing input parameters. He further states that the simulation results are descriptive only and do not provide an idea of optimal solutions (2005).

The simulation I have used in this thesis is not intended to provide optimal solutions but it does show the results of a “what if” analysis where the transition probabilities are changed from the currently estimated values, to the optimized values. The simulation results compare the model forecasts by determining the estimated level of risk involved using each set of transition variables.

### 3. Optimization Models

The goal of an optimization model is to either minimize or maximize an objective function by adjusting a set of decision variables. This capability is a major component of the variable flow model.

The workforce optimization models discussed by Wang are linear programming, integer programming, goal programming, and dynamic programming models.

Linear and integer programming only differ in that the resultant decision variables in the integer model are integers. Both models are used to find a single objective function. I use linear programming in my models to optimize the number of accessions and also the transition rates. The model is only able to solve one of these functions at a time.

Goal programming varies by being able to solve multiple objective functions simultaneously, but only as a minimization function. There is a requirement for the objective functions to be weighted dependent on their importance, which would be determined by the relevant decision makers within the organization. While I designed the model with the ability to weight the objective function, I have not used goal programming as the MT situation does not require the combined optimization of both the accessions and the transition probabilities, as the number of accessions is currently at its upper limit.

Wang describes dynamic programming as being able to use the theory associated with linear programming to break a problem down into multiple levels and conduct an optimization at each level before applying those results to the next stage in order to give an optimized final objective (2005). I have not incorporated this function into the variable flow model at this time. The potential exists for this capability to be included as it would provide the benefit of being able to optimize transition probabilities each year as opposed to using one set of constant optimized transition probabilities over the period of the optimization.

### 4. Markov Chain Models

Wang describes Markov chain theory as a mathematical tool used to investigate dynamic behaviours of a system in a discrete-time stochastic process. It is well suited to modelling the manpower structures found in workforce, financial, and health service systems (2005).

Markov models are limited, however, in that they do not have the mathematical techniques required to conduct an optimization (Wang, 2005). That may be the case, but in this thesis, the Markov framework provides the ideal baseline from which the other variable flow model capabilities function, e.g., the Markov model generates the transition probabilities used in the accession optimization, and also the forecast formula which is used in both the accession and transition probability optimizations.

According to Wang, for a Markov model to be valid, the population within the workforce must be able to be placed in classes which are both exhaustive and mutually exclusive (2005). In this thesis, the workforce is classified by rank. Individuals can only be classified in one rank at any time, and there are a finite number of ranks they can be classed in, meaning these criteria for a Markov model are met.

Wang discusses advantages of a Markov chain model that are relevant to this thesis. This type of model can be used to assist in development of promotion and recruitment policies, is suitable for forecasting manpower requirements, and is currently used for workforce modelling in the Australian Army and the United States Air Force (2005).

Wang states that another potential limitation of a Markov model is that a large sample, greater than 100 in each class, is required to give stable transition probability estimates (2005). The only class, or rank, that does not meet this criterion is E09. For this reason, the E09 rank is left out of objective function calculations in the optimization scenarios.

For sample sizes smaller than 100, Wang (2005) recommends using a computer simulation model.

In summary, Wang's (2005) paper discusses four major types of quantitative models used for workforce planning. I have used three of these models as components of the variable flow model. Markov modelling allows the calculation of transition probabilities of personnel through each state in the MT system, LP optimization provides both the optimal number of accessions and the optimal transition probabilities to aim towards, and simulation allows the level of risk involved using the current or optimized transition probabilities to be determined.

## B. MARKOV MODELLING APPROACHES

There have been no studies published that research end strength planning for the RAN. There have, however, been numerous other studies into military workforce planning that have used a Markov modelling approach. This section identifies and discusses two such studies that have been used to contribute to the content of this thesis. Licari (2013) wrote his paper with the purpose of developing a model that will be utilized as a tool for accession and end strength planning in the Selected Marine Corps Reserve (SMCR). Gass, Collins, Meinhardt, Lemon, & Gillette (1988) produced a paper titled "OR Practice—The Army Manpower Long-Range Planning System." Both of these papers are relevant to this thesis as they discuss end strength planning in a military environment, and use modelling concepts which may aid in the reduction of manpower shortages in the MT branch.

Licari (2013) justifies the use of a Markov model in his thesis by reviewing work from Bartholomew (1991), Sales (1971), and Kalamatianou (1987), and comparing their relevance to the U.S. Marine Corps. As an example, Sales (1971) uses a Markov model to simulate civil service grade flows, which Licari uses as justification for his choice of using a Markov model based on the similarities between the structure of the civil service and the Marine Corps Reserve Component. Licari's justification also applies to the use of a Markov model to represent the transition through the MT branch.

Licari uses a fixed inventory Markov model, which allows for changes in the number of accessions between periods, with the required end strength numbers both known and constant. The fixed inventory model is employed in my thesis when conducting the accession optimization.

In Licari's model, there are only two movement options for personnel—to the next state or to the absorbing state (attrition). This differs from my model where personnel can stay in the same state, move up one state, move up two states, or attrite.

Another difference between Licari's thesis and this thesis is that Licari does not conduct any optimization to determine the best number of accessions to meet end strength requirements. His analysis uses estimated transition probabilities to determine what the end strength will be with the current rate of accessions. This is the same as the base case scenario described in Data and Methodology. This thesis includes both accession optimization and also transition probability optimization.

Licari acknowledges the main limitation of his model is that some states will have insufficient numbers to give a statistically significant transition probability. That issue is addressed in this thesis by increasing the time interval to one year and reducing the number of states to allow an appropriate sample size in each state.

The Manpower Long-Range Planning System (MLRPS) contains several functions which I have employed in the variable flow model. According to Gass et al., the MLRPS was designed to project future end strength, and determine optimal transition rates and accession values to meet defined end strength values (1988). These are the same capabilities that the variable flow model delivers.

The MLRPS is divided into a data processing subsystem, a flow model subsystem and an optimization subsystem. This is a logical sequence of calculations that I have included in the variable flow model, but in a less formal manner. The data processing subsystem results in the output of the transition probabilities which populate the Markov model in the flow model subsystem. The required optimization is then carried out by minimizing a weighted objective function.

The MLRPS is more complex than the variable flow model as it has to cater for a force structure that is much larger than just the MT branch that my model focuses on. I have taken the approach that each branch within the organization should be treated separately, and have its own forecasts. Doing this will minimize the processing requirements of the model and allow the use of a basic optimization tool such as Excel.

The other significant difference between the MLRPS and my model is that the variable flow model incorporates simulation to estimate the level of risk involved in the model forecasts, due to variance in the estimated transition probabilities. This is an added capability that could assist planners in explaining their forecast results to the decision makers.

# III. DATA AND METHODOLOGY

The data for this thesis was provided by the RAN and is used to formulate a Markov chain model representing the transition through the MT branch of enlisted sailors. The Markov model forms the basis of the variable flow model. This section discusses the data provided by the RAN and how it will be used in the variable flow model to provide various forecasts of end strength based on different optimization parameters.

## A. DATA

The data set for this thesis was obtained from the RAN's Directorate of Workforce Modelling, Forecasting, and Analysis (DWMFA). The following data was used to build the model:

- Attrition—The attrition database contains information on all 3,543 personnel who separated from the MT branch of the RAN from July 2002, to May 2016. All personnel have been assigned a random identifier number, corresponding to their related data on the other databases. Relevant data on all attrites includes their date of separation and rank at time of separation.

- Promotion—The promotion database lists all 10,400 promotions in the MT branch from July 2002, to May 2016. The relevant data used is the identifier number, rank promoted to, rank promoted from, and date of promotion.

- Hires—The Hires database contains information on all 3,505 accessions into the MT branch from July 2002, until May 2016. The relevant data used is the identifier number, the date of hire, and the rank on hiring.

- Historical End strength—The End Strength database contains 200,051 observations of the end strength of the MT branch for each month from June 2006 to May 2016. From the beginning of the period until November 2012, the data included all personnel of the rank E02 and above, whether they were classified as trained or untrained. After November 2012, the database only includes personnel considered as trained. The relevant data used is the identifier number and the rank of each member at the end of each month. Whether an individual is trained or untrained is irrelevant in this model.

- Under Training End Strength—From December 2012, to May 2016, the monthly end strength of MT personnel under training is captured on a separate database. The relevant information provided by this database is the total number of each rank under training at the end of each month.
- Required End Strength—DWMFA provided the required end strength of ranks E03, E05, E06, and E08 for each year up until 2030. The E03 rank is made up of sailors who are classified as either trained or untrained. The number of E03 required is the number of trained E03. (2016)

## B. METHODOLOGY

The methodology section explains how the data was used to generate the required matrices, how to interpret these matrices, and the matrix formulae used to forecast end strength.

### 1. Data Selection

The first step in building the Markov model for this thesis was to decide whether the observed time interval would be monthly or yearly. The decision was made to use a one year time interval as this increases the sample size for promotions, attrites, and hires and also removes the possibility of seasonality affecting the estimation of the transition probabilities. Seasonality could be an issue as people are more likely to attrite at certain times within a year and certain months have no hires or promotions at all.

### 2. Annual Transition Probabilities

The data was sorted into yearly intervals based on the Australian financial year (FY) which runs from July 1 to June 30. Figure 1 shows an example of the resultant matrix containing actual end strength numbers for FY 14-15.

| FY 14-15 | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | E00-E01 | E02 | E03 | E05 | E06 | E08 | E09 | ATTRITES | Total |
| E00-E01 | 202 | 197 | 26 | 0 | 0 | 0 | 0 | 23 | 448 |
| E02 | 0 | 0 | 156 | 0 | 0 | 0 | 0 | 9 | 165 |
| E03 | 0 | 0 | 656 | 55 | 0 | 0 | 0 | 95 | 806 |
| E05 | 0 | 0 | 0 | 303 | 21 | 0 | 0 | 50 | **374** |
| E06 | 0 | 0 | 0 | 0 | 222 | 15 | 0 | 13 | 250 |
| E08 | 0 | 0 | 0 | 0 | 0 | 182 | 5 | 10 | 197 |
| E09 | 0 | 0 | 0 | 0 | 0 | 0 | 38 | 1 | 39 |
| | 202 | 197 | 838 | 358 | 243 | 197 | 43 | 201 | 2279 |

Figure 1. MT End Strength FY 14–15 (Adapted from DWMFA, 2016).

Using the highlighted row as an example, in FY 14–15 there were 806 personnel who either started the year in the E03 rank or were hired straight into that rank during the year. At the end of the year, 656 were still E03, 55 had been promoted to E05, and 95 had attrited.

The highlighted E06 column indicates that at the end of FY 14–15, there were 253 members at the rank of E06, 232 of those had started the year at that rank and 21 had been promoted from E05.

The values in the bottom row make up the stock vector that will be used in the calculation for forecast end strength.

The transition probabilities are calculated from the end strength matrix by dividing the end strength number in each cell by the row total. The results from FY 14–15 are provided as an example in Figure 2.

| FY 14-15 | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | E00-E01 | E02 | E03 | E05 | E06 | E08 | E09 | ATTRITES |
| E00-E01 | 0.451 | 0.440 | 0.058 | 0.000 | 0.000 | 0.000 | 0.000 | 0.051 |
| E02 | 0.000 | 0.000 | 0.945 | 0.000 | 0.000 | 0.000 | 0.000 | 0.055 |
| E03 | 0.000 | 0.000 | 0.814 | 0.068 | 0.000 | 0.000 | 0.000 | 0.118 |
| E05 | 0.000 | 0.000 | 0.000 | 0.810 | 0.056 | 0.000 | 0.000 | 0.134 |
| E06 | 0.000 | 0.000 | 0.000 | 0.000 | 0.888 | 0.060 | 0.000 | 0.052 |
| E08 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.924 | 0.025 | 0.051 |
| E09 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.974 | 0.026 |

Figure 2. MT transition probabilities FY 14–15

Again using the highlighted row as an example, the probability of an individual starting the period as an E02 and finishing as E03 is 0.945. The remaining personnel either attrite or stay as E02. Personnel generally do not stay as E02, unless they have disciplinary or physical issues, as there is a non-discretionary promotion from E02 to E03 after 12 months.

### 3. Estimation of Transition and Distribution Probabilities

The transition probabilities to be used in the base case and accession optimization scenarios are estimated using transition data from the latest two years of observations.

This range has been chosen due to a change in career continuum introduced in 2010 and the follow on effect it has had on attrition rates. The data show that attrition rates, specifically on the important E03, E05, and E06 ranks, have been gradually declining (see Appendix). By selecting only the last two years of data, the higher attrition rates resulting from an inferior prior continuum are not included in the model. Figure 3 shows the estimated transition values used in the model.

| | E00-E01 | E02 | E03 | E05 | E06 | E08 | E09 | ATTRITES |
|---|---|---|---|---|---|---|---|---|
| **E00-E01** | 0.426 | 0.458 | 0.056 | 0.000 | 0.000 | 0.000 | 0.000 | 0.060 |
| **E02** | 0.000 | 0.060 | 0.901 | 0.000 | 0.000 | 0.000 | 0.000 | 0.038 |
| **E03** | 0.000 | 0.000 | 0.797 | 0.085 | 0.000 | 0.000 | 0.000 | 0.119 |
| **E05** | 0.000 | 0.000 | 0.000 | 0.817 | 0.081 | 0.000 | 0.000 | 0.102 |
| **E06** | 0.000 | 0.000 | 0.000 | 0.000 | 0.888 | 0.067 | 0.000 | 0.045 |
| **E08** | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.946 | 0.013 | 0.041 |
| **E09** | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.987 | 0.013 |

Figure 3. Estimated Transition Matrix

The distribution of new hires is also estimated using data from the last two observed years. This calculation involves adding the number of accessions entering each rank and then dividing by the total of all hires. The result, as shown in Table 1, is a percentage distribution of all new hires.

Table 1. Estimated Distribution Probabilities (Adapted from DWMFA, 2016).

| **Total Hires 14–15 15–16** | | **% allocation** |
|---|---|---|
| **E00-E01** | 418 | 0.917 |
| **E02** | 0 | 0.000 |
| **E03** | 13 | 0.029 |
| **E05** | 11 | 0.024 |
| **E06** | 10 | 0.022 |
| **E08** | 4 | 0.009 |
| **E09** | 0 | 0.000 |
| **SUM** | 456 | 1 |

## 4. Matrix Calculations

The formula for forecasting the stock vector in the next period, *n(t)*, as given in Guerry and De Feyter (2009), is:

$$n(t) = n(t-1)P(t-1) + R(t)r(t-1).$$

where:

$n(t-1)$ will be a $(1\times 7)$ matrix with the number of stock in each state at the end of the previous time interval.

$P(t-1)$ will be a $(7\times 7)$ matrix with the estimated transition probabilities in the previous time interval. These probabilities remain constant through all years forecast.

$R(t)$ is a single value representing the total number of accessions in the current time period.

$r(t-1)$ will be a $(1\times 7)$ matrix with the estimated distribution probabilities, constant across all years.

The result will be a $(1\times 7)$ matrix containing the forecast end strength of each rank at the end of year $t$. During analysis, it was found that this formula did not accurately model the MT continuum. The problem discovered was that the transition matrix is not applied to new hires. In reality, a proportion of new hires who enter as either E00 or E01, transition to E02 within a year. To fix this problem, the stock vector forecast formula is changed as follows:

$$n(t) = n(t-1)P(t-1) + (R(t)r(t-1))P(t-1).$$

This change also reflects the Markov assumption that the model is memoryless (Guerry and De Feyter, 2009). Personnel entering the workforce into a given rank have the same probability of transitioning as a member who has been in that rank for an extended period.

The end strength stock vector from FY 15–16 and the distribution matrix are constant through all the scenarios and use the following values:

$$n(t-1) = \begin{pmatrix}165 & 222 & 842 & 382 & 255 & 208 & 42\end{pmatrix}$$

$$r(t-1) = \begin{pmatrix}0.917 & 0.0 & 0.029 & 0.024 & 0.022 & 0.009 & 0.0\end{pmatrix}$$

The transition matrix uses the values in Figure 3 for the base case and accession optimization scenarios. The values for the transition matrix are found by optimization in the other scenarios. The value of $R(t)$ is 228 in all scenarios except scenario 2, where the accession number is calculated by the optimization process.

## C. SCENARIOS

I have run two types of optimization scenarios, accession optimization and transition probability optimization, as well as a non-optimizing base case which forecasts future end strength based on the current hiring and transition rates. I have changed parameters in each optimization scenario to not only show the variation in forecasts but also to demonstrate the capabilities of the model. Table 2 lists the number and title of each scenario conducted.

Table 2. Table of Scenarios

| Scenario Number | Scenario Title |
|---|---|
| 1 | Base Case |
| 2 | Accession Optimization—Adjust Upper Accession Constraint |
| 3A | Transition Probability Optimization Short Term Strategy |
| 3B | Transition Probability Optimization Mid-Term Strategy |
| 3C | Transition Probability Optimization Mid-Term Strategy—Adjust Objective Function by Weighting Over and Under End Strength |
| 3D | Transition Probability Optimization Mid-Term Strategy—Adjust Objective Function by Weighting End Strength Difference Based on Rank |

Unless otherwise stated, the objective function in each optimization scenario is the sum of the absolute difference between the forecast and desired end strength of E03, E05, E06, and E08 personnel each year over the next seven years. Figure 4 is an example of how the E05 component of the objective function is calculated. The figures highlighted in blue are the cumulative difference added each year until the seventh year. This figure is then added to the corresponding E03, E06, and E08 values to give the final objective function figure for minimization.

| | | | **E05** | | |
|---|---|---|---|---|---|
| **FY** | **n(0)** | Forecast | Desired | | |
| **16-17** | **n(1)** | 388.501 | 443 | 54.499 | 54.499 |
| **17-18** | **n(2)** | 398.348 | 434 | 35.652 | 90.151 |
| **18-19** | **n(3)** | 407.137 | 430 | 22.863 | 113.014 |
| **19-20** | **n(4)** | 414.535 | 469 | 54.465 | 167.479 |
| **20-21** | **n(5)** | 420.643 | 477 | 56.357 | 223.836 |
| **21-22** | **n(6)** | 425.641 | 452 | 26.359 | 250.195 |
| **22-23** | **n(7)** | 429.712 | 468 | 38.288 | 288.483 |

Figure 4. Example Objective Function Calculation.

The reasoning for calculating the end strength difference across seven years is that it takes, on average, seven years for a new hire to reach the critical rank of E05, therefore any period shorter than this would have a limited effect on the E05 deficit. Even though the model forecasts out to fourteen years, I have not included the final seven years in the objective function as these projections are too far in the future and are subject to too much uncertainty.

There is a mix of trained and untrained personnel in the E03 rank but the desired end strength data provided by DWMFA is only for the number of trained personnel. There was no data available on the required end strength of untrained, or under training E03 sailors. The E03 end strength is too important to leave out of any optimization so I have used the trained end strength requirement plus a variable to simulate the total required end strength of the E03 rank. I will set this variable at 200 for the optimizations.

### 1. Base Case

This scenario did not involve an optimization. The model was run using a constant yearly accession value, $R(t)$, of 228, with the distribution percentages as shown in Table 1. The transition probabilities used are those shown in Figure 3. The purpose of this scenario is to determine if future end strength requirements can be met with the current fixed recruiting policy and transition rates, and if not, what is the forecast difference?

### 2. Accession Optimization Scenario

For the last 3 years of data observations, the RAN has hired exactly 228 personnel per year, indicating that this is the maximum that the training continuum can handle. The purpose of this scenario is to simulate an increase in training capability for the MT branch, which will in turn allow a greater number of accessions per year. The model allows the accession limits to be adjusted and in this case I have simulated an increase from the current base case accession number by 30%. This raises the yearly accession number limit from 228 to 296.

The decision variable to be optimized is $R(t)$, the number of hires in a given year. The constraints are the upper bound of 296 hires per year and a lower bound of 200 per year. There is no change in the objective function previously defined.

### 3. Transition Probability Optimization Scenarios

The other component of the variable flow model is the ability to optimize the transition matrix without changing the current hiring policy.

This capability is necessary if the forecast from the base case scenario shows that the current transition rates will not, over time, result in an effective reduction of the end strength deficit. If the RAN is currently hiring the maximum number possible per year, their only remaining solution to achieve the optimal mix of personnel between ranks is to adjust the probabilities in the transition matrix.

It is important to note that the results of these transition probability optimizations are not a forecast of what may happen given the current situation. These optimizations give optimal transition probabilities (targets) that an organization must work towards achieving in order to reach the forecast end strength that the variable flow model provides. Identifying methods to bring transition probabilities in line with the forecast is outside the scope of this thesis.

The only data required to conduct this optimization is the latest known end strength and the future end strength requirements. It is beneficial to have a history of transition data to compare optimization results against. It is also necessary, for this case study, to know the minimum time spent in each rank before a member is eligible for promotion. This information is necessary to set constraints for the model.

The decision variables used to minimize the objective function in this model are all the transition probabilities within the transition matrix.

The constraints are more numerous, and a touch more complicated, than in the accession optimization model. The first constraint requires the generation of a fundamental matrix.

The fundamental matrix, *S*, is calculated using the following formula:

$$S = (I - P(t))^{-1}$$

where:

*I* is a (7 x 7) Identity Matrix, and

*P(t)* is the (7 x 7) transition matrix.

The derivation of the fundamental matrix is described in Ross (2000). The main diagonal of a fundamental matrix gives the average time spent in that rank. For example, Figure 5 shows the Fundamental matrix which corresponds to the transition probabilities in the base case transition matrix. From this data, one can tell that, on average, an MT sailor spends 4.917 years as an E03. The values for E08 and E09 are exaggerated due to a combination of the limited time frame and low attrition rates in those ranks over the last two years of observations. The off diagonal values are not relevant for the optimization.

| **S-Matrix** | | | Using 2 year model | | | | |
|---|---|---|---|---|---|---|---|
| | **E00-E01** | **E02** | **E03** | **E05** | **E06** | **E08** | **E09** |
| **E00-E01** | 1.743 | 0.849 | 4.214 | 1.952 | 1.421 | 1.744 | 1.727 |
| **E02** | 0.000 | 1.064 | 4.684 | 2.170 | 1.580 | 1.939 | 1.919 |
| **E03** | 0.000 | 0.000 | 4.917 | 2.277 | 1.658 | 2.035 | 2.015 |
| **E05** | 0.000 | 0.000 | 0.000 | 5.465 | 3.980 | 4.885 | 4.835 |
| **E06** | 0.000 | 0.000 | 0.000 | 0.000 | 8.963 | 11.001 | 10.889 |
| **E08** | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 18.394 | 18.207 |
| **E09** | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 74.750 |

Figure 5. Example of a Fundamental Matrix.

A constraint matrix was used with the minimum time in rank set along the main diagonal as shown in Figure 6. These figures are adjustable as required.

| | | | **Constraint Matrix** | | | | |
|---|---|---|---|---|---|---|---|
| | **E00-E01** | **E02** | **E03** | **E05** | **E06** | **E08** | **E09** |
| **E00-E01** | 1.600 | | | | | | |
| **E02** | | 1.000 | | | | | |
| **E03** | | | 4.000 | | | | |
| **E05** | | | | 4.000 | | | |
| **E06** | | | | | 4.000 | | |
| **E08** | | | | | | 4.000 | |
| **E09** | | | | | | | |

Figure 6. Minimum Constraint Matrix for Transition Probability Optimization.

The decision variables must be calculated so that they do not result in a value lower than the one given in the constraint matrix. There is no value for E09 as they have nowhere to promote to.

Another constraint matrix lists maximum values for time in rank. This is necessary for E00-E01 and E02 as these ranks have a ceiling time. E00-E01 are under training and their time in that rank only varies based on availability of courses and holiday periods. As previously mentioned, all E02 personnel are promoted to E03 automatically after 12 months, with the minor exception of those on disciplinary or physical limitations. The remaining ranks were given a realistic maximum value as shown in Figure 7. These values are also easily adjustable within the model.

| | Maximum Constraint Matrix | | | | | | |
|---|---|---|---|---|---|---|---|
| | **E00-E01** | **E02** | **E03** | **E05** | **E06** | **E08** | **E09** |
| **E00-E01** | 1.800 | | | | | | |
| **E02** | | 1.100 | | | | | |
| **E03** | | | 8.000 | | | | |
| **E05** | | | | 8.000 | | | |
| **E06** | | | | | 10.000 | | |
| **E08** | | | | | | 10.000 | |
| **E09** | | | | | | | 10.000 |

Figure 7. Maximum Constraint Matrix for Transition Probability Optimization.

More constraints were added to ensure the probabilities in each row of the transition matrix sum to 1, there can be no negative numbers, and the transition probabilities, including the attrition rate, should be realistic. To achieve a realistic solution, I have set the minimum value for attritions, to the minimum value for that cell over the last 6 years. All the matrix cells which cannot be transitioned into are constrained to a value of zero, e.g., an individual cannot transition from E03 to E09 in one time interval so that cell is set to zero.

The objective functions for the following scenarios vary in each and are explained in the scenario description. The number of accessions, and their percentage distribution, remain constant through these scenarios.

#### *a. Scenario 3A—Transition Probability Optimization Short Term Strategy*

It is reasonable to assume that the RAN will want to reduce the end strength deficit in as short a time as possible. For this reason, I have set the objective function to be the absolute value of the combined forecast end strength difference of E03, E05, E06, and E08 after one year. The model will still forecast out to fourteen years, allowing future effects of the short term strategy to be observed and compared to the results of the next scenarios.

#### *b. Scenario 3B—Transition Probability Optimization Mid-Term Strategy*

This scenario will optimize the transition probabilities using the same objective function as scenario 2, where the absolute value of the difference between forecast and desired end strength is added across E03, E05, E06, and E08 each year for the next seven years.

By employing a mid-term strategy, the model will be able to allow for future increases or decreases in desired end strength (within the next seven years). This is important and relevant as the RAN is acquiring new Ships within the next seven years causing the required end strength numbers to increase.

#### *c. Scenario 3C—Transition Probability Optimization Mid-Term Strategy Adjust Objective Function by Weighting Over and Under End Strength*

The model has the capability to weight the objective function based on organizational preferences. This scenario uses the same decision variables and constraints as the previous scenario. The objective function is composed of the same end strength differentials across E03-E08, but I have assumed that the RAN would prefer end strength forecasts to be over rather than under so have weighted the difference more heavily when the forecast falls below the desired level. This will cause the model to work harder at bringing a below forecast towards the desired end strength rather than an overage forecast of the same magnitude.

Because a large majority of the forecasts in the seven years being optimized will be negative, the weighting needs to take into account how negative the forecast is, as opposed to whether it is simply under or over the desired amount.

For the purpose of this scenario, the weightings were set identically for each rank with the values shown in Figure 8.

| Forecast | Weight |
|---|---|
| < =-50 | 0.5 |
| >-50, <=-25 | 0.3 |
| >-25, <0 | 0.15 |
| >=0 | 0.05 |

Figure 8. Objective Function Weighting for Forecast Difference from Desired End Strength.

#### d. *Scenario 3D— Transition Probability Optimization Mid-Term Strategy Adjust Objective Function by Weighting End Strength Difference Based on Rank*

This scenario uses the same decision variables and constraints as the previous scenario. Here, the objective function is weighted based on the perceived importance of rank. The weighting of the difference in end strength dependent on rank is difficult to determine. The weighting could be determined by the perceived value each rank brings to the RAN, or by the average number of essential positions in a Ship's complement, or by the biggest current deficit in forecast end strength. The model can be adjusted to cater for any of these situations.

For the purpose of this scenario, I have weighted the difference in end strength based on the average individual dollar value that each rank is worth, determined by their average annual pay grade. Figure 9 shows an example of how the objective function is weighted for this scenario.

| E03 | | E05 | | E06 | | E08 | |
|---|---|---|---|---|---|---|---|
| $pa | 60542 | $pa | 66888 | $pa | 79438 | $pa | 93639 |
| Weight | 0.201 | Weight | 0.223 | Weight | 0.264 | Weight | 0.312 |
| 7 Year Difference | 344.798 | | 185.077 | | 188.260 | | 97.053 |
| Weighted Difference | 69.465 | | 41.195 | | 49.766 | | 30.242 |
| | | | | | | | |
| **Sum of differences (7 years):** | | | | | 815.188 | | |
| **Sum of weighted differences:** | | | | | 190.668 | **Objective Function** | |

Figure 9. Example of Weighting Objective Function by Rank.

## D. SUMMARY

This chapter has discussed the data used as the basis for the variable flow model and how the data were used to generate transition and distribution probabilities for building the Markov chain component of the model.

I have also explained the formula used for making forecasts and finally, described the scenarios used to demonstrate the model’s optimization capabilities.

The following chapter will discuss the results of the different scenarios.

# IV. RESULTS AND RISK ANALYSIS

This chapter lays out the results of the base case scenario, the accession optimization scenario, and the transition probability optimization scenarios described in the previous chapter. For each scenario, I have provided, at a minimum, the forecast end strength results for ranks E05, E06, and E08 for the next 14 years, out to FY 29–30. Each rank has a column showing the difference from desired end strength. Forecasts below desired end strength are highlighted red, while forecasts greater than desired are highlighted green.

I have also included steady state tables to show where the forecast values will settle using the differing transition probabilities in each scenario. These values are compared against the latest known desired end strength values (FY 29–30). The optimization targets are in the intermediate future, before steady state solutions are reached. This is appropriate because the optimal force structure changes over time, and reaching that force structure can take a very long time. However, the impact of the solutions on the steady state variance is also presented for each scenario, to emphasize the fact that optimal transitional solutions are not the same as optimal steady state solutions. Presenting the steady state solutions also provides another basis for the comparison of solutions, besides that which is being optimized.

## A. BASE CASE RESULTS

The base case results are intended to show what the model predicts will occur if the status quo is maintained. No optimization was performed and no parameters were changed in this scenario. The transition probabilities in Figure 3 were used. Figure 10 shows the current hiring and distribution average per year and Figure 11 shows the forecast end strength for the given ranks and the associated forecast deficit/surplus.

| Forecast Accessions | | E00-E01 | E02 | E03 | E05 | E06 | E08 | E09 | |
|---|---|---|---|---|---|---|---|---|---|
| | | r | | | | | | | |
| n(0) | R | 0.916667 | 0 | 0.028509 | 0.024123 | 0.02193 | 0.008772 | 0 | **FY** |
| n(1) | 228 | 209 | 0 | 7 | 6 | 5 | 2 | 0 | **16-17** |
| n(2) | 228 | 209 | 0 | 7 | 6 | 5 | 2 | 0 | **17-18** |
| n(3) | 228 | 209 | 0 | 7 | 6 | 5 | 2 | 0 | **18-19** |
| n(4) | 228 | 209 | 0 | 7 | 6 | 5 | 2 | 0 | **19-20** |
| n(5) | 228 | 209 | 0 | 7 | 6 | 5 | 2 | 0 | **20-21** |
| n(6) | 228 | 209 | 0 | 7 | 6 | 5 | 2 | 0 | **21-22** |
| n(7) | 228 | 209 | 0 | 7 | 6 | 5 | 2 | 0 | **22-23** |
| n(8) | 228 | 209 | 0 | 7 | 6 | 5 | 2 | 0 | **23-24** |
| n(9) | 228 | 209 | 0 | 7 | 6 | 5 | 2 | 0 | **24-25** |
| n(10) | 228 | 209 | 0 | 7 | 6 | 5 | 2 | 0 | **25-26** |
| n(11) | 228 | 209 | 0 | 7 | 6 | 5 | 2 | 0 | **26-27** |
| n(12) | 228 | 209 | 0 | 7 | 6 | 5 | 2 | 0 | **27-28** |
| n(13) | 228 | 209 | 0 | 7 | 6 | 5 | 2 | 0 | **28-29** |
| n(14) | 228 | 209 | 0 | 7 | 6 | 5 | 2 | 0 | **29-30** |

Figure 10. Scenario 1—Base Case Hires and Distribution of Personnel

| | E03 | | | E05 | | | E06 | | | E08 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| FY | Forecast | Desired | +/- | Forecast | Desired | +/- | Forecast | Desired | +/- | Forecast | Desired | +/- |
| 16-17 | 897 | 1011 | -114 | 389 | 443 | -54 | 262 | 305 | -43 | 216 | 234 | -18 |
| 17-18 | 907 | 1006 | -99 | 398 | 434 | -36 | 270 | 306 | -36 | 224 | 229 | -5 |
| 18-19 | 910 | 999 | -89 | 407 | 430 | -23 | 277 | 306 | -29 | 232 | 230 | 2 |
| 19-20 | 911 | 1066 | -155 | 415 | 469 | -54 | 284 | 326 | -42 | 240 | 243 | -3 |
| 20-21 | 912 | 1076 | -164 | 421 | 477 | -56 | 291 | 330 | -39 | 248 | 241 | 7 |
| 21-22 | 912 | 1034 | -122 | 427 | 452 | -25 | 298 | 319 | -21 | 256 | 230 | 26 |
| 22-23 | 912 | 1055 | -143 | 431 | 468 | -37 | 304 | 327 | -23 | 264 | 241 | 23 |
| 23-24 | 912 | 1081 | -169 | 434 | 478 | -44 | 310 | 333 | -23 | 273 | 249 | 24 |
| 24-25 | 912 | 1073 | -161 | 437 | 478 | -41 | 315 | 333 | -18 | 281 | 245 | 36 |
| 25-26 | 912 | 1064 | -152 | 439 | 470 | -31 | 321 | 322 | -1 | 289 | 236 | 53 |
| 26-27 | 912 | 1073 | -161 | 441 | 474 | -33 | 325 | 323 | 2 | 297 | 237 | 60 |
| 27-28 | 912 | 1080 | -168 | 443 | 478 | -35 | 330 | 326 | 4 | 304 | 238 | 66 |
| 28-29 | 912 | 1073 | -161 | 444 | 474 | -30 | 334 | 323 | 11 | 312 | 237 | 75 |
| 29-30 | 912 | 1080 | -168 | 445 | 478 | -33 | 338 | 326 | 12 | 320 | 238 | 82 |

Figure 11. Scenario 1—Base Case End Strength Forecast.

The objective function value, representing the combined total forecast number of overages and shortages across the ranks of E03 through E08, over seven years, is 1491.

It is evident from the forecast in Figure 11 that by maintaining the current approach, the E08 deficit will be eliminated in the next 4 years, the E06 deficit will be eliminated in 11 years, but the E03 and E05 deficits will continue. These figures indicate that the RAN does not currently have the correct mix of transition rates for personnel to flow efficiently through the system. Table 3 shows the steady state of the system using the base case parameters, proving that the current mix needs to be adjusted. The steady state figures were calculated by extending the model's forecast past FY 29–30 until a steady state was reached. The desired values are the end strength requirements in FY 29–30.

Table 3. Scenario 1 Steady State Analysis Results

| | E00-E01 | E02 | E03 | E05 | E06 | E08 | E09 |
|---|---|---|---|---|---|---|---|
| **Initial Value** | 165 | 222 | 842 | 382 | 255 | 208 | 42 |
| **Desired** | - | - | 1080 | 478 | 326 | 238 | - |
| **Steady State** | 155.315 | 177.407 | 911.711 | 449.817 | 371.385 | 496.769 | 490.595 |

The steady state analysis shows that with the current transition probabilities and hiring rate, the E05 rank will never reach the desired end strength, the E06 rank will settle at a limit 14% above the desired level, and E08 will eventually settle at 108% of the desired end strength. I do not have the desired end strength data for E00-E01, E02, all of the E03 (trained and untrained), and E09, but it can be safely assumed that the steady state figure for E09 is excessive and the figure for the total of E03 is inadequate.

The following scenario results show the effect of addressing this problem through, firstly, changing the hiring rate, and then by optimizing the transition rates while keeping the hiring rate constant.

## B. ACCESSION OPTIMIZATION RESULTS

For this scenario, the only parameter change from the base case scenario was the 30% increase in the accessions limit. The reasoning for this increase was to attempt to eliminate the end strength deficit by adding more people to the system, while still using the same transition rates. This scenario is unlikely to occur in reality unless the RAN commits to a large increase in training capabilities.

The results from the optimization are shown in Figures 12 and 13. Figure 12 shows the optimized number of accessions and their distribution, while Figure 13 shows the forecast end strength.

| Forecast Accessions | | E00-E01 | E02 | E03 | E05 | E06 | E08 | E09 | |
|---|---|---|---|---|---|---|---|---|---|
| | | r | | | | | | | |
| n(0) | R | 0.916667 | 0 | 0.028509 | 0.024123 | 0.02193 | 0.008772 | 0 | FY |
| n(1) | 296 | 272 | 0 | 8 | 7 | 7 | 3 | 0 | 16-17 |
| n(2) | 296 | 272 | 0 | 8 | 7 | 7 | 3 | 0 | 17-18 |
| n(3) | 296 | 272 | 0 | 8 | 7 | 7 | 3 | 0 | 18-19 |
| n(4) | 296 | 272 | 0 | 8 | 7 | 7 | 3 | 0 | 19-20 |
| n(5) | 214 | 196 | 0 | 6 | 5 | 5 | 2 | 0 | 20-21 |
| n(6) | 296 | 272 | 0 | 8 | 7 | 7 | 3 | 0 | 21-22 |
| n(7) | 296 | 272 | 0 | 8 | 7 | 7 | 3 | 0 | 22-23 |
| n(8) | 296 | 272 | 0 | 8 | 7 | 7 | 3 | 0 | 23-24 |
| n(9) | 200 | 183 | 0 | 6 | 5 | 4 | 2 | 0 | 24-25 |
| n(10) | 292 | 268 | 0 | 8 | 7 | 6 | 3 | 0 | 25-26 |
| n(11) | 288 | 264 | 0 | 8 | 7 | 6 | 3 | 0 | 26-27 |
| n(12) | 248 | 227 | 0 | 7 | 6 | 5 | 2 | 0 | 27-28 |
| n(13) | 296 | 272 | 0 | 8 | 7 | 7 | 3 | 0 | 28-29 |
| n(14) | 200 | 183 | 0 | 6 | 5 | 4 | 2 | 0 | 29-30 |

Figure 12. Scenario 2— Hires and Distribution.

The 30% increase in accessions raises the limit from 228 in the base case scenario, to 296 hires per year. The results in Figure 12 show that the limit constrains the number of hires six of the seven years of the optimization. The accessions after FY 22–23 are not factored into the optimization, but are still relevant as they are realistic numbers which help show the forecast effect that the increased hiring policy will have on end strength after the initial seven year optimization period

| FY | E03 Forecast | E03 Desired | E03 +/- | E05 Forecast | E05 Desired | E05 +/- | E06 Forecast | E06 Desired | E06 +/- | E08 Forecast | E08 Desired | E08 +/- |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 16-17 | 902 | 1011 | -109 | 390 | 443 | -53 | 264 | 305 | -41 | 217 | 234 | -17 |
| 17-18 | 943 | 1006 | -63 | 402 | 434 | -32 | 273 | 306 | -33 | 225 | 229 | -4 |
| 18-19 | 984 | 999 | -15 | 415 | 430 | -15 | 281 | 306 | -25 | 234 | 230 | 4 |
| 19-20 | 1022 | 1066 | -44 | 429 | 469 | -40 | 290 | 326 | -36 | 243 | 243 | 0 |
| 20-21 | 1048 | 1076 | -28 | 442 | 477 | -35 | 297 | 330 | -33 | 251 | 241 | 10 |
| 21-22 | 1042 | 1034 | 8 | 456 | 452 | 4 | 306 | 319 | -13 | 260 | 230 | 30 |
| 22-23 | 1055 | 1055 | 0 | 468 | 468 | 0 | 315 | 327 | -12 | 269 | 241 | 28 |
| 23-24 | 1074 | 1081 | -7 | 478 | 478 | 0 | 324 | 333 | -9 | 279 | 249 | 30 |
| 24-25 | 1087 | 1073 | 14 | 486 | 478 | 8 | 331 | 333 | -2 | 287 | 245 | 42 |
| 25-26 | 1067 | 1064 | 3 | 496 | 470 | 26 | 340 | 322 | 18 | 297 | 236 | 61 |
| 26-27 | 1069 | 1073 | -4 | 502 | 474 | 28 | 349 | 323 | 26 | 306 | 237 | 69 |
| 27-28 | 1077 | 1080 | -3 | 506 | 478 | 28 | 356 | 326 | 30 | 315 | 238 | 77 |
| 28-29 | 1074 | 1073 | 1 | 511 | 474 | 37 | 364 | 323 | 41 | 325 | 237 | 88 |
| 29-30 | 1078 | 1080 | -2 | 513 | 478 | 35 | 369 | 326 | 43 | 333 | 238 | 95 |

Figure 13. Scenario 2—End Strength Forecast.

The major differences between the two scenarios, measured at FY 22–23, are listed in Table 4. The values in the E03-E08 columns represent the forecast variation from the desired end strength at that year.

Table 4. Differences Between Scenario 1 and 2

| Scenario | Total Hires | E03 | E05 | E06 | E08 | Objective Function |
|---|---|---|---|---|---|---|
| **1** | 1596 | -143 | -37 | -23 | 23 | 1491 |
| **2** | 2072 | 0 | 0 | -12 | 28 | 736 |

It is evident that this strategy is effective in reducing shortages in a seven year timespan. This strategy will, however, have the same issues as scenario 1, only magnified now that the number of hires has increased. To get the correct mix, the RAN would have to adjust the number of promotions, e.g., in FY 22–23, they would promote less people from E06 to E08 to reduce the forecast shortage in E06 and the overage of E08. This is effectively changing the transition probabilities.

The purpose of the following optimizations is to provide transition probabilities that, if achieved, will give a better mix of personnel in the appropriate ranks.

## C. TRANSITION PROBABILITY OPTIMIZATION SCENARIOS

There are four transition probability optimizations, the results of which highlight the difference in taking a short-term versus a mid-term manning strategy, and also the effect of adjusting the objective function. The results of each will be compared against each other and also the base case results.

### 1. Scenario 3A

This scenario is designed to reduce the end strength deficits in the quickest time possible. The transition probabilities are optimized by minimizing an objective function calculated only over the first year. The objective function is calculated by summing the absolute difference between the forecast and desired E03, E05, E06, and E08 end strengths in FY 16–17.

Figure 14 shows the resultant transition matrix, Figure 15 shows the associated fundamental matrix, and Figure 16 shows the end strength forecast using the optimized transition probabilities.

| | E00-E01 | E02 | E03 | E05 | E06 | E08 | E09 | ATTRITES |
|---|---|---|---|---|---|---|---|---|
| **E00-E01** | **0.394** | **0.497** | **0.058** | 0.000 | 0.000 | 0.000 | 0.000 | **0.051** |
| **E02** | 0.000 | **0.000** | **0.985** | 0.000 | 0.000 | 0.000 | 0.000 | **0.015** |
| **E03** | 0.000 | 0.000 | **0.819** | **0.076** | 0.000 | 0.000 | 0.000 | **0.105** |
| **E05** | 0.000 | 0.000 | 0.000 | **0.836** | **0.097** | 0.000 | 0.000 | **0.067** |
| **E06** | 0.000 | 0.000 | 0.000 | 0.000 | **0.879** | **0.084** | 0.000 | **0.037** |
| **E08** | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | **0.900** | **0.035** | **0.065** |
| **E09** | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | **0.750** | **0.250** |

Figure 14. Scenario 3A—Optimized Transition Matrix.

| | E00-E01 | E02 | E03 | E05 | E06 | E08 | E09 |
|---|---|---|---|---|---|---|---|
| **E00-E01** | **1.650** | 0.820 | 4.987 | 2.324 | 1.859 | 1.566 | 0.217 |
| **E02** | 0.000 | **1.000** | 5.440 | 2.535 | 2.028 | 1.709 | 0.237 |
| **E03** | 0.000 | 0.000 | **5.523** | 2.574 | 2.059 | 1.735 | 0.241 |
| **E05** | 0.000 | 0.000 | 0.000 | **6.102** | 4.881 | 4.113 | 0.571 |
| **E06** | 0.000 | 0.000 | 0.000 | 0.000 | **8.244** | 6.947 | 0.964 |
| **E08** | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | **10.000** | 1.388 |
| **E09** | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | **4.000** |

Figure 15. Scenario 3A—Fundamental Matrix Corresponding to Optimized Transition Matrix in Figure 14.

The fundamental matrix shows that the average transition time for an E03 to E05 has increased from 4.9 to 5.5 years, and E05 to E06 has increased from 5.4 to 6.1 years. These figures alone are difficult to interpret without looking at the effect they have on the forecast end strength.

| | E03 | | | E05 | | | E06 | | | E08 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **FY** | Forecast | Desired | +/- | Forecast | Desired | +/- | Forecast | Desired | +/- | Forecast | Desired | +/- |
| **16-17** | 935 | 1011 | -76 | 389 | 443 | -54 | 266 | 305 | -39 | 211 | 234 | -23 |
| **17-18** | 975 | 1006 | -31 | 402 | 434 | -32 | 276 | 306 | -30 | 214 | 229 | -15 |
| **18-19** | 998 | 999 | -1 | 415 | 430 | -15 | 287 | 306 | -19 | 219 | 230 | -11 |
| **19-20** | 1014 | 1066 | -52 | 429 | 469 | -40 | 297 | 326 | -29 | 223 | 243 | -20 |
| **20-21** | 1025 | 1076 | -51 | 441 | 477 | -36 | 308 | 330 | -22 | 228 | 241 | -13 |
| **21-22** | 1034 | 1034 | 0 | 452 | 452 | 0 | 318 | 319 | -1 | 233 | 230 | 3 |
| **22-23** | 1041 | 1055 | -14 | 462 | 468 | -6 | 328 | 327 | 1 | 239 | 241 | -2 |
| **23-24** | 1047 | 1081 | -34 | 471 | 478 | -7 | 338 | 333 | 5 | 245 | 249 | -4 |
| **24-25** | 1051 | 1073 | -22 | 479 | 478 | 1 | 348 | 333 | 15 | 251 | 245 | 6 |
| **25-26** | 1055 | 1064 | -9 | 486 | 470 | 16 | 357 | 322 | 35 | 258 | 236 | 22 |
| **26-27** | 1058 | 1073 | -15 | 492 | 474 | 18 | 366 | 323 | 43 | 264 | 237 | 27 |
| **27-28** | 1060 | 1080 | -20 | 497 | 478 | 19 | 374 | 326 | 48 | 271 | 238 | 33 |
| **28-29** | 1062 | 1073 | -11 | 502 | 474 | 28 | 382 | 323 | 59 | 277 | 237 | 40 |
| **29-30** | 1064 | 1080 | -16 | 506 | 478 | 28 | 389 | 326 | 63 | 284 | 238 | 46 |

Figure 16. Scenario 3A—Forecast End Strength Using Transition Matrix in Figure 14.

Figure 16 shows that using the short term strategy will provide a small benefit in E03 and E06 ranks in FY 16–17, when compared to the base case.

It is interesting to note that using the transition matrix in Figure 14, all ranks are forecast to be either close to, or exactly on the desired end strength at the six year mark.

To maintain the correct mix of personnel from that point would require a readjustment of the transition matrix as the forecasts diverge away from the desired end strength.

As the number of accessions is constant again, I have also measured the model's forecasts against the desired steady state. Table 5 compares the steady state results from this example to scenario 1. Again, the desired results are the end strength requirements at FY 29–30. It is apparent that this model better meets the organizational requirements than Scenario 1, but if these transition probabilities are maintained, the MT branch will not have an adequate number of E05, and will have a large overage of E06 and E08.

Table 5. Scenario 3A Steady State Comparison

| | **E00-E01** | **E02** | **E03** | **E05** | **E06** | **E08** | **E09** |
|---|---|---|---|---|---|---|---|
| **Initial Value** | 165 | 222 | 842 | 382 | 255 | 208 | 42 |
| **Desired** | - | - | 1080 | 478 | 326 | 238 | - |
| **Steady State Scenario 1** | 155.315 | 177.407 | 911.711 | 449.817 | 371.385 | 496.769 | 490.595 |
| **Steady State Scenario 3A** | 135.850 | 156.558 | 1142.707 | 414.102 | 412.059 | 291.988 | 40.798 |

### 2. Scenario 3B

This scenario uses a longer term approach to minimizing end strength, with the aim of getting the MT branch back on track in seven years. The objective function differs from the previous scenario as the absolute difference in forecast and desired end strength, across E03 to E08, is now summed over the next seven years instead of one year.

Figure 17 shows the optimized transition matrix, Figure 18 shows the associated fundamental matrix, and Figure 19 shows the forecast end strength.

| | E00-E01 | E02 | E03 | E05 | E06 | E08 | E09 | ATTRITES |
|---|---|---|---|---|---|---|---|---|
| **E00-E01** | **0.394** | **0.497** | **0.058** | 0.000 | 0.000 | 0.000 | 0.000 | **0.051** |
| **E02** | 0.000 | **0.000** | **0.985** | 0.000 | 0.000 | 0.000 | 0.000 | **0.015** |
| **E03** | 0.000 | 0.000 | **0.819** | **0.076** | 0.000 | 0.000 | 0.000 | **0.105** |
| **E05** | 0.000 | 0.000 | 0.000 | **0.839** | **0.095** | 0.000 | 0.000 | **0.067** |
| **E06** | 0.000 | 0.000 | 0.000 | 0.000 | **0.881** | **0.082** | 0.000 | **0.037** |
| **E08** | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | **0.900** | **0.035** | **0.065** |
| **E09** | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | **0.750** | **0.250** |

Figure 17. Scenario 3B—Optimized Transition Matrix.

| | E00-E01 | E02 | E03 | E05 | E06 | E08 | E09 |
|---|---|---|---|---|---|---|---|
| **E00-E01** | **1.650** | 0.820 | 4.988 | 2.359 | 1.871 | 1.538 | 0.213 |
| **E02** | 0.000 | **1.000** | 5.441 | 2.573 | 2.041 | 1.677 | 0.233 |
| **E03** | 0.000 | 0.000 | **5.524** | 2.612 | 2.072 | 1.703 | 0.236 |
| **E05** | 0.000 | 0.000 | 0.000 | **6.195** | 4.913 | 4.039 | 0.561 |
| **E06** | 0.000 | 0.000 | 0.000 | 0.000 | **8.386** | 6.894 | 0.957 |
| **E08** | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | **10.000** | 1.388 |
| **E09** | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | **4.000** |

Figure 18. Scenario 3B—Fundamental Matrix Corresponding to Optimized Transition Matrix in Figure 17.

The fundamental matrix in this scenario only has minor differences to the short term strategy fundamental matrix. The E05 and E06 average time in rank are slightly longer in this example. The effect this has on the end strength forecast can be seen in Figure 19.

| | E03 | | | E05 | | | E06 | | | E08 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| FY | Forecast | Desired | +/- | Forecast | Desired | +/- | Forecast | Desired | +/- | Forecast | Desired | +/- |
| 16-17 | 935 | 1011 | -76 | 390 | 443 | -53 | 266 | 305 | -39 | 210 | 234 | -24 |
| 17-18 | 975 | 1006 | -31 | 403 | 434 | -31 | 276 | 306 | -30 | 213 | 229 | -16 |
| 18-19 | 998 | 999 | -1 | 418 | 430 | -12 | 286 | 306 | -20 | 217 | 230 | -13 |
| 19-20 | 1014 | 1066 | -52 | 432 | 469 | -37 | 296 | 326 | -30 | 221 | 243 | -22 |
| 20-21 | 1025 | 1076 | -51 | 444 | 477 | -33 | 307 | 330 | -23 | 225 | 241 | -16 |
| 21-22 | 1034 | 1034 | 0 | 456 | 452 | 4 | 317 | 319 | -2 | 230 | 230 | 0 |
| 22-23 | 1041 | 1055 | -14 | 467 | 468 | -1 | 327 | 327 | 0 | 236 | 241 | -5 |
| 23-24 | 1047 | 1081 | -34 | 476 | 478 | -2 | 337 | 333 | 4 | 241 | 249 | -8 |
| 24-25 | 1051 | 1073 | -22 | 484 | 478 | 6 | 347 | 333 | 14 | 247 | 245 | 2 |
| 25-26 | 1055 | 1064 | -9 | 491 | 470 | 21 | 356 | 322 | 34 | 253 | 236 | 17 |
| 26-27 | 1058 | 1073 | -15 | 498 | 474 | 24 | 365 | 323 | 42 | 259 | 237 | 22 |
| 27-28 | 1061 | 1080 | -19 | 503 | 478 | 25 | 374 | 326 | 48 | 266 | 238 | 28 |
| 28-29 | 1063 | 1073 | -10 | 508 | 474 | 34 | 382 | 323 | 59 | 272 | 237 | 35 |
| 29-30 | 1064 | 1080 | -16 | 512 | 478 | 34 | 389 | 326 | 63 | 278 | 238 | 40 |

Figure 19. Scenario 3B—Forecast End Strength Using Transition Matrix in Figure 17.

Again, there is very little difference between this forecast and the previous scenario. At the six year mark, the personnel mix is adequate but would require a readjustment of the transition rates to slow down the overage increase in the E05-E08 ranks. Table 6 shows that the longer term strategy (3B) steady state, results in large overages in ranks E05 and above, suggesting this model is not adequate to solve the MT problem. Using this model will provide a better situation than maintaining the status quo as the E03 and E05 ranks will be better populated.

Table 6. Steady State Comparison of Scenarios 3A and 3B

| | **E00-E01** | **E02** | **E03** | **E05** | **E06** | **E08** | **E09** |
|---|---|---|---|---|---|---|---|
| **Initial Value** | 165 | 222 | 842 | 382 | 255 | 208 | 42 |
| **Desired** | - | - | 1080 | 478 | 326 | 238 | - |
| **Steady State Scenario 1** | 155.315 | 177.407 | 911.711 | 449.817 | 371.385 | 496.769 | 490.595 |
| **Steady State Scenario 3A** | 135.850 | 156.558 | 1142.707 | 414.102 | 412.059 | 291.988 | 40.798 |
| **Steady State Scenario 3B** | 135.850 | 171.282 | 1071.855 | 538.511 | 468.394 | 407.155 | 56.781 |

### 3. Scenario 3C

This scenario uses the same long term approach as Scenario 3B, with the same constraints and decision variables. The difference is that the objective function is now weighted to simulate a request from the RAN that they would prefer forecasts to be over rather than under the desired end strength. The weightings applied to the difference between forecast and desired end strength will be those shown in Figure 8. Using these figures gives preference to reducing the largest deficits.

Figure 20 shows the optimized transition matrix, Figure 21 shows the associated fundamental matrix, and Figure 22 shows the forecast end strength.

| | E00-E01 | E02 | E03 | E05 | E06 | E08 | E09 | ATTRITES |
|---|---|---|---|---|---|---|---|---|
| **E00-E01** | **0.394** | **0.497** | **0.058** | 0.000 | 0.000 | 0.000 | 0.000 | **0.051** |
| **E02** | 0.000 | **0.000** | **0.985** | 0.000 | 0.000 | 0.000 | 0.000 | **0.015** |
| **E03** | 0.000 | 0.000 | **0.820** | **0.075** | 0.000 | 0.000 | 0.000 | **0.105** |
| **E05** | 0.000 | 0.000 | 0.000 | **0.840** | **0.093** | 0.000 | 0.000 | **0.067** |
| **E06** | 0.000 | 0.000 | 0.000 | 0.000 | **0.884** | **0.079** | 0.000 | **0.037** |
| **E08** | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | **0.900** | **0.035** | **0.065** |
| **E09** | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | **0.750** | **0.250** |

Figure 20. Scenario 3C—Optimized Transition Matrix.

| | E00-E01 | E02 | E03 | E05 | E06 | E08 | E09 |
|---|---|---|---|---|---|---|---|
| **E00-E01** | **1.650** | 0.820 | 5.029 | 2.359 | 1.884 | 1.489 | 0.207 |
| **E02** | 0.000 | **1.000** | 5.486 | 2.573 | 2.055 | 1.625 | 0.225 |
| **E03** | 0.000 | 0.000 | **5.570** | 2.613 | 2.086 | 1.650 | 0.229 |
| **E05** | 0.000 | 0.000 | 0.000 | **6.267** | 5.004 | 3.957 | 0.549 |
| **E06** | 0.000 | 0.000 | 0.000 | 0.000 | **8.612** | 6.810 | 0.945 |
| **E08** | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | **10.000** | 1.388 |
| **E09** | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | **4.000** |

Figure 21. Scenario 3C—Fundamental Matrix Corresponding to Optimized Transition Matrix in Figure 20

The fundamental matrix in this example shows a trend towards increasing the average time in rank for E03, E05, and E06 when compared against Scenario 3B. The effect of this is seen in Figure 22.

| | E03 | | | E05 | | | E06 | | | E08 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| FY | Forecast | Desired | +/- | Forecast | Desired | +/- | Forecast | Desired | +/- | Forecast | Desired | +/- |
| 16-17 | 937 | 1011 | -74 | 389 | 443 | -54 | 266 | 305 | -39 | 210 | 234 | -24 |
| 17-18 | 977 | 1006 | -29 | 402 | 434 | -32 | 276 | 306 | -30 | 212 | 229 | -17 |
| 18-19 | 1002 | 999 | 3 | 416 | 430 | -14 | 286 | 306 | -20 | 215 | 230 | -15 |
| 19-20 | 1018 | 1066 | -48 | 430 | 469 | -39 | 296 | 326 | -30 | 218 | 243 | -25 |
| 20-21 | 1030 | 1076 | -46 | 443 | 477 | -34 | 307 | 330 | -23 | 222 | 241 | -19 |
| 21-22 | 1040 | 1034 | 6 | 454 | 452 | 2 | 317 | 319 | -2 | 226 | 230 | -4 |
| 22-23 | 1047 | 1055 | -8 | 465 | 468 | -3 | 327 | 327 | 0 | 231 | 241 | -10 |
| 23-24 | 1053 | 1081 | -28 | 474 | 478 | -4 | 337 | 333 | 4 | 236 | 249 | -13 |
| 24-25 | 1058 | 1073 | -15 | 482 | 478 | 4 | 347 | 333 | 14 | 241 | 245 | -4 |
| 25-26 | 1062 | 1064 | -2 | 490 | 470 | 20 | 356 | 322 | 34 | 247 | 236 | 11 |
| 26-27 | 1066 | 1073 | -7 | 496 | 474 | 22 | 365 | 323 | 42 | 252 | 237 | 15 |
| 27-28 | 1068 | 1080 | -12 | 502 | 478 | 24 | 374 | 326 | 48 | 258 | 238 | 20 |
| 28-29 | 1071 | 1073 | -2 | 507 | 474 | 33 | 382 | 323 | 59 | 264 | 237 | 27 |
| 29-30 | 1072 | 1080 | -8 | 511 | 478 | 33 | 390 | 326 | 64 | 270 | 238 | 32 |

Figure 22. Scenario 3C—Forecast End Strength Using Transition Matrix in Figure 20.

Figure 22 demonstrates that the model works towards reducing the largest negative differences. These differences exist predominantly in the E03 and E05 ranks and the model addresses this by increasing the average time spent in each rank. When compared to the Scenario 3B forecast, there is a noticeable reduction in the E03 deficit.

Table 7 shows that the steady state values for this model indicate that in the long run, it will provide a better result for E03, E05, E08, and E09 than both the previous model and the base case scenario.

Table 7. Steady State Comparison of Scenarios 3B and 3C

| | E00-E01 | E02 | E03 | E05 | E06 | E08 | E09 |
|---|---|---|---|---|---|---|---|
| **Initial Value** | 165 | 222 | 842 | 382 | 255 | 208 | 42 |
| **Desired** | - | - | 1080 | 478 | 326 | 238 | - |
| **Steady State Scenario 1** | 155.315 | 177.407 | 911.711 | 449.817 | 371.385 | 496.769 | 490.595 |
| **Steady State Scenario 3B** | 135.850 | 171.282 | 1071.855 | 538.511 | 468.394 | 407.155 | 56.781 |
| **Steady State Scenario 3C** | 135.850 | 171.282 | 1080.806 | 538.983 | 472.793 | 395.838 | 55.210 |

### 4. Scenario 3D

The final scenario uses the same constraints and variables as the previous two examples. The objective function in this model weights the forecast end strength differences based on rank. It is assumed an E08 is more important than an E06, who is in turn more important than E05, and so on. This logic is simply based on the fact that the higher the rank, the higher the pay. Empty E08 positions are costing the RAN more than empty E03 positions.

Figure 23 shows the optimized transition matrix, Figure 24 shows the associated fundamental matrix, and Figure 25 shows the forecast end strength.

| | E00-E01 | E02 | E03 | E05 | E06 | E08 | E09 | ATTRITES |
|---|---|---|---|---|---|---|---|---|
| **E00-E01** | **0.394** | **0.497** | **0.058** | 0.000 | 0.000 | 0.000 | 0.000 | **0.051** |
| **E02** | 0.000 | **0.000** | **0.985** | 0.000 | 0.000 | 0.000 | 0.000 | **0.015** |
| **E03** | 0.000 | 0.000 | **0.819** | **0.077** | 0.000 | 0.000 | 0.000 | **0.105** |
| **E05** | 0.000 | 0.000 | 0.000 | **0.836** | **0.097** | 0.000 | 0.000 | **0.067** |
| **E06** | 0.000 | 0.000 | 0.000 | 0.000 | **0.877** | **0.086** | 0.000 | **0.037** |
| **E08** | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | **0.900** | **0.029** | **0.071** |
| **E09** | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | **0.750** | **0.250** |

Figure 23. Scenario 3C—Optimized Transition Matrix.

| | E00-E01 | E02 | E03 | E05 | E06 | E08 | E09 |
|---|---|---|---|---|---|---|---|
| **E00-E01** | **1.650** | 0.820 | 4.982 | 2.321 | 1.845 | 1.578 | 0.182 |
| **E02** | 0.000 | **1.000** | 5.435 | 2.532 | 2.013 | 1.721 | 0.198 |
| **E03** | 0.000 | 0.000 | **5.518** | 2.571 | 2.043 | 1.748 | 0.201 |
| **E05** | 0.000 | 0.000 | 0.000 | **6.088** | 4.839 | 4.138 | 0.477 |
| **E06** | 0.000 | 0.000 | 0.000 | 0.000 | **8.159** | 6.978 | 0.804 |
| **E08** | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | **10.000** | 1.152 |
| **E09** | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | **4.000** |

Figure 24. Scenario 3C—Fundamental Matrix Corresponding to Optimized Transition Matrix in Figure 23

As would be expected, when compared to the fundamental matrix in Scenario 3C, the average time in rank for E03, E05, and E06 has decreased. This allows a quicker transition to E08. The average time in rank for E08 could not go any higher as it is at the upper limit applied to the model.

| | E03 | | | E05 | | | E06 | | | E08 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| FY | Forecast | Desired | +/- | Forecast | Desired | +/- | Forecast | Desired | +/- | Forecast | Desired | +/- |
| 16-17 | 935 | 1011 | -76 | 389 | 443 | -54 | 266 | 305 | -39 | 211 | 234 | -23 |
| 17-18 | 975 | 1006 | -31 | 402 | 434 | -32 | 276 | 306 | -30 | 215 | 229 | -14 |
| 18-19 | 998 | 999 | -1 | 415 | 430 | -15 | 286 | 306 | -20 | 219 | 230 | -11 |
| 19-20 | 1013 | 1066 | -53 | 429 | 469 | -40 | 297 | 326 | -29 | 224 | 243 | -19 |
| 20-21 | 1025 | 1076 | -51 | 441 | 477 | -36 | 307 | 330 | -23 | 229 | 241 | -12 |
| 21-22 | 1033 | 1034 | -1 | 452 | 452 | 0 | 317 | 319 | -2 | 235 | 230 | 5 |
| 22-23 | 1040 | 1055 | -15 | 462 | 468 | -6 | 327 | 327 | 0 | 241 | 241 | 0 |
| 23-24 | 1046 | 1081 | -35 | 471 | 478 | -7 | 337 | 333 | 4 | 247 | 249 | -2 |
| 24-25 | 1050 | 1073 | -23 | 479 | 478 | 1 | 346 | 333 | 13 | 253 | 245 | 8 |
| 25-26 | 1054 | 1064 | -10 | 485 | 470 | 15 | 356 | 322 | 34 | 260 | 236 | 24 |
| 26-27 | 1057 | 1073 | -16 | 491 | 474 | 17 | 364 | 323 | 41 | 266 | 237 | 29 |
| 27-28 | 1060 | 1080 | -20 | 497 | 478 | 19 | 372 | 326 | 46 | 273 | 238 | 35 |
| 28-29 | 1062 | 1073 | -11 | 501 | 474 | 27 | 380 | 323 | 57 | 280 | 237 | 43 |
| 29-30 | 1063 | 1080 | -17 | 505 | 478 | 27 | 387 | 326 | 61 | 287 | 238 | 49 |

Figure 25. Scenario 3C—Forecast End Strength Using Transition Matrix in Figure 23.

The transition rates result in personnel moving through the system to fill the weighted E08 positions at the expense of E03 and E05 positions. Figure 25 shows a noticeable difference from the previous scenario forecasts. At the seven year mark, the deficit of E03 has increased from 8 to 15, and E05 deficit has increased from 3 to 6. The E08 difference has changed from a 10 person deficit to a zero deficit. Because the optimization is conducted over seven years, the effects of propping up the E08 rank in the mid-term are seen out to FY 29–30 and beyond. Table 8 shows that this model results in a decrease in E03, E05, E06, and E09 steady state compared to Scenario 3C. The E03 end state is the most concerning as it is below the desired amount.

Table 8. Steady State Comparison of Scenarios 3C and 3D

| | E00-E01 | E02 | E03 | E05 | E06 | E08 | E09 |
|---|---|---|---|---|---|---|---|
| **Initial Value** | 165 | 222 | 842 | 382 | 255 | 208 | 42 |
| **Desired** | - | - | 1080 | 478 | 326 | 238 | - |
| **Steady State Scenario 1** | 155.315 | 177.407 | 911.711 | 449.817 | 371.385 | 496.769 | 490.595 |
| **Steady State Scenario 3C** | 135.850 | 171.282 | 1080.806 | 538.983 | 472.793 | 395.838 | 55.210 |
| **Steady State Scenario 3D** | 135.849 | 171.282 | 1070.694 | 529.851 | 461.295 | 416.789 | 48.228 |

## D. RISK ANALYSIS

As this model uses estimation to generate a transition matrix, it is necessary to determine the level of risk involved in making the forecasts due to error in the estimated probabilities.

For the risk analysis, I have used simulation to calculate numerous objective functions, based on transition probabilities varying around the estimated values.

The process to generate the transition probabilities uses the following logic:

1. Generate a random variable, $X_{ij} \sim Binom\ (P = P_{ij}, n = 100)$, where, $P_{ij}$ is the transition probability at row *i*, column *j* of the transition matrix.
2. Generate the transition probability to be used in the simulation, $P'_{ij}$, by calculating $P'_{ij} = X_{ij} / 100$.

This process is repeated to generate probabilities along the main diagonal, as well as the attrition rates, until a transition matrix is completed. The values of $P'_{ij}$ are constrained so that the sum of all probabilities in each row, including attrites, is equal to 1. The forecasting formula is then applied to each randomly generated transition matrix and the resulting forecasts are compared to the desired end strength values to determine the objective function value. The objective function was calculated using the difference between forecast and desired end strength in E05, E06, and E08. The simulation repeats this process 900,000 times to give the results in Figures 26–29.

The objective function is calculated over one year for the first two simulations and over seven years for the final two.

Figure 26 shows the results of varying the estimated transition probabilities in the base case scenario. The mean objective function value is 115.89 and there is a 5% probability that the objective function will be greater than 166.68.

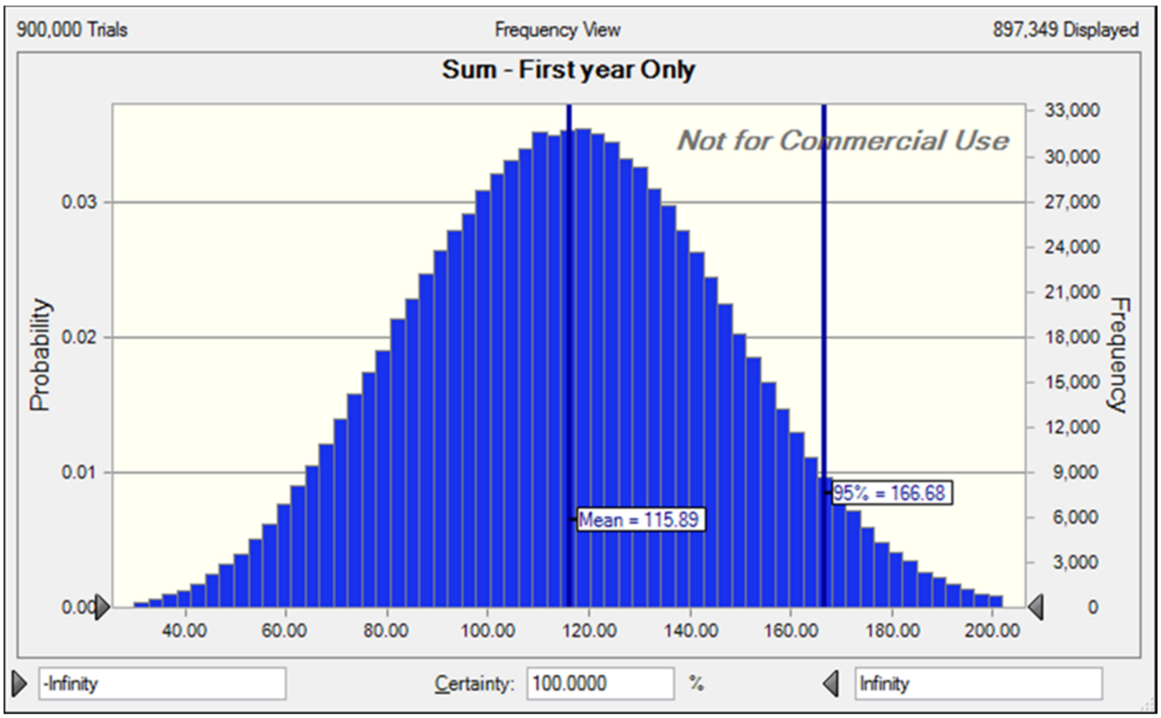


Figure 26. Base Case Transition Probabilities Risk Analysis Over One Year

Figure 27 uses the same objective function as Figure 26, but the transition probabilities used are the result of the transition probability optimization from Scenario 3A. The simulation results show that the mean objective function has decreased to 82.67 with a 5% probability that the value would be greater than 104.02 (which is still less than the mean of the base case example). This clearly illustrates that using the optimized transition rates is superior compared to maintaining the status quo, and reduces not only the expected deviation, but the risk that the deviation is excessively large.

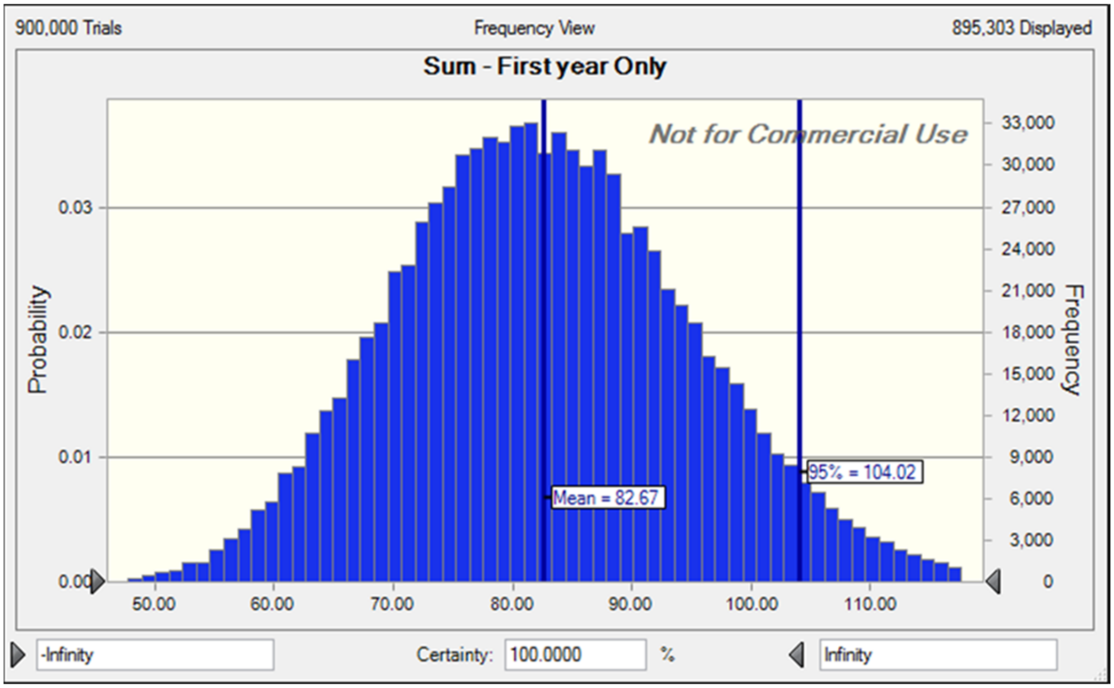


Figure 27. Optimized Transition Probabilities Risk Analysis Over One Year

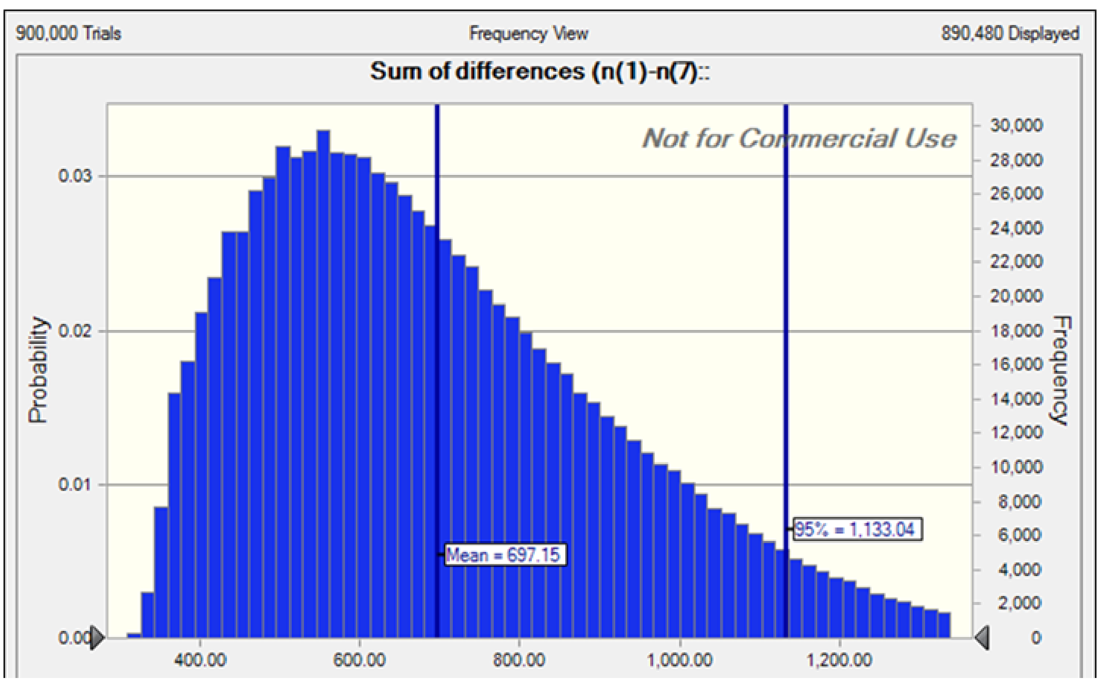


Figure 28. Base Case Risk Analysis Over Seven Years

The next simulation varies the base case transition probabilities, but the objective function is now totaled over seven years. Figure 28 shows that the mean absolute difference from desired end strength is 697.15 people, with a 5% probability that the value could be greater than 1,133.04.

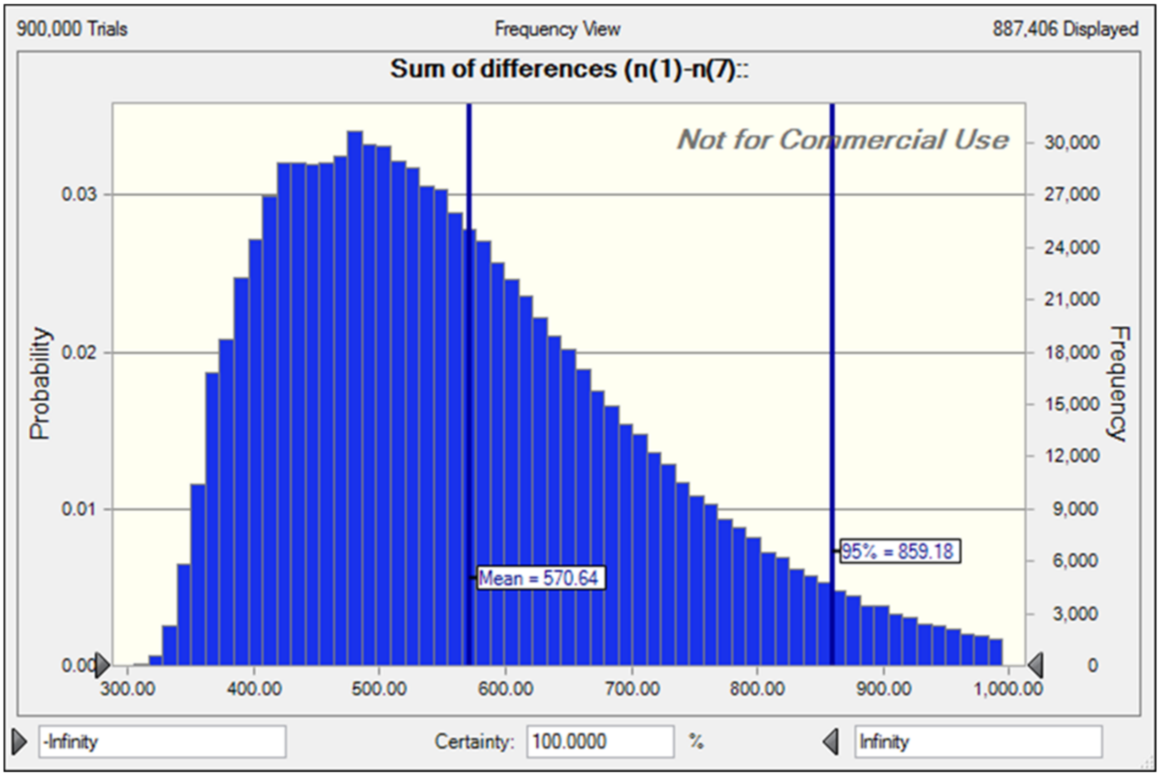


Figure 29. Optimized Transition Probabilities Over Seven Years

The final simulation, shown in Figure 29, calculates the objective function over seven years by varying the transition probabilities optimized in Scenario 3B. The mean difference from desired end strength has decreased to 570.64 and there is a 5% probability that the difference could be greater than 859.18, compared to 1,133.04 with the same probability in the previous simulation.

These numbers again indicate that there is less risk involved in adopting the optimized transition probabilities than there is by maintaining the status quo.

## E. SUMMARY

The results demonstrate that if the RAN continues with its current hiring policy, with the same transition rates through the system, then they will never reach the desired end strength requirements for the E03 and E05 ranks. A 30% increase in the number of hires will, in six years' time, give the MT branch the right number of people, but with the wrong rank mix. It is unlikely that the RAN will be able to cater for such an increase in hires, so this scenario is not likely to solve their problem.

Each transition rate optimization shows that if the RAN were able to achieve the target transition probabilities, then in six years' time, using the current hiring numbers and distribution, they would be able to meet the desired end strength in each rank.

The results also show that once the desired end strength is achieved, the transition rates need to be reassessed to maintain the correct mix of personnel.

The risk analysis confirms that using the optimized transition probabilities is a better option than maintaining the status quo, as there is less risk of higher end strength differences involved

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# V. LIMITATIONS, RECOMMENDATIONS AND CONCLUSION

This section will discuss the limitations I have found with the variable flow model in its current format and provide recommendations for both improving the model and for future research associated with the variable flow modelling process. Finally, I will conclude the thesis by wrapping up the major points and achievements.

## A. LIMITATIONS

I have observed several limitations with this initial iteration of the variable flow model. The following factors have affected the reliability and accuracy of the model's forecasts:

- Insufficient Data—The data provided was of excellent quality, but to provide an effective forecast I needed more data in regards to the required number of personnel under training, specifically at the E03 rank. In the model I have used an estimation of this number which is easily adjustable once the actual figures are known.

- MT Branch Sub-Categories—The MT branch is actually divided into four different specialities once sailors reach the E03 rank. Data identifying which speciality each sailor belongs to is not yet available The model currently forecasts the branch as a whole, but would be more effective if each specialty was treated individually.

- Microsoft Excel—Excel was inconsistent in the values it reached when optimizing the transition probabilities. Generally, I would need to clear all the previous decision variables before running a new optimization, or run an optimization using a different objective function before going back and trying again. A production version of the prototype model here should perhaps use a better Non-Linear Programming engine.

- Attrition Rates—The estimation of the optimal attrition rate is the only model variable that the RAN will have limited control over. If this probability cannot be achieved then the rest of the forecast is affected. The risk analysis presented in Chapter IV, Section D addresses this limitation, or at least quantifies it.

- Implementation of Model Forecasts—The transition rate optimization component of the model is specifically concerned with generating target values. Methods to achieve these targets are not discussed in this thesis.

## B. RECOMMENDATIONS

This model has provided several forecasts to aid the RAN workforce planners in reducing the MT branch manpower deficit. My recommendation is that if the current hiring policy is maintained, then the planners should consider the transition probability optimization forecasts which all show that the deficits can be considerably reduced within six years if the RAN is able to alter the current transition rates to the forecast rates.

To address the limitations previously stated, I also recommend the following:

- The RAN updates the model to include up to date end strength requirements for E03 under training.
- For data analysis purposes, the RAN should divide end strength requirements for the branch into requirements for each specialty within the branch.
- Excel Solver is an easy and available optimization tool, but I recommend further investigation into alternative software packages that may offer more reliability and consistency.
- In this model I have used the minimum attrition rates for each rank over the last six years of observations as the lower attrition constraint in each optimization. I recommend that for a "worst case scenario" the maximum attrition rates should be used as a minimum constraint. I also recommend further research into methods of limiting the attrition rate so that forecast rates can be achieved.
- Further research needs to be conducted to identify methods for converging the current transition rates to the optimal transition rates, e.g., is there an optimal way to change the current rates on a year by year basis to achieve end strength sooner? As discussed in the Literature Review, dynamic programming may be able to address this problem.

## C. CONCLUSION

The purpose of this thesis is to develop a model to assist military manpower planners in meeting prescribed end strength requirements. In order to achieve this, I proposed two main research questions, and a series of subordinate questions, that needed to be answered:

1. Can a Markov Modelling approach aid in solving end strength problems in the RAN?

i) Can a Markov Model be built to predict MT end strength?

ii) Can a linear program be developed to optimize accessions and transition rates in the MT ranks?

iii) What are some of the shortcomings of the Markov model approach, as applied to the MT branch?

2. Can Simulation be used to estimate the risk of falling below or above end strength targets?

i) Can Simulation be used to estimate end strength target risks in the MT ranks?

ii) What data are required to use simulation to estimate such risks?

The results of Scenarios 1 and 2 demonstrate that the variable flow model, which is based on Markov chain theory, is able to predict MT end strength using estimated transition probabilities, a given number of accessions, and a known stock vector. Scenario 2 also demonstrates the model's ability to optimize the number of accessions in order to reduce the end strength deficit.

Scenarios 3A through D all demonstrate that the model is capable of optimizing the transition rates dependent on the differing objective functions. The transition probability optimization scenarios show that by changing the transition rates, the end strength deficits can be eliminated in six years.

The limitations of a Markov model are discussed in Chapter II, with the main limitation being that the model does not include feedback for any cause and effect relationships. This limitation was acknowledged and accepted as the alternative models were too complex for the scope of this thesis.

The main element of risk assessed in this thesis was the error involved in estimating the transition probabilities. The risk analysis in Chapter IV demonstrated that simulation could be used to estimate the effect on the objective function of the transition probabilities varying within the deviation limits, e.g., using Scenario 1, the simulation assessed that there was a 5% probability that the objective function would be greater than 166, compared to the 115 that was forecast. If there is a particular limit to overages/ deficits that is of concern, the risk of reaching that limit is easy to assess with this model.

So long as the Binomial Distribution can be reasonably assumed for transition numbers, the data required to effectively run a simulation are the same data required to estimate the transition probabilities. In this case, those data are promotions, attritions, hires, and end strength observed over the last six financial years.

The variable flow model has been able to meet the requirements of the research questions, and is at a state where it can be modified to suit any military organization wishing to identify their optimal transition rates.

# APPENDIX. HISTORICAL TRANSITION MATRICES

| FY 10-11 | E00-E01 | E02 | E03 | E05 | E06 | E08 | E09 | ATTRITES | Total |
|---|---|---|---|---|---|---|---|---|---|
| E00-E01 | 0.428 | 0.485 | 0.005 | 0.000 | 0.000 | 0.000 | 0.000 | 0.082 | 425 |
| E02 | 0.000 | 0.116 | 0.869 | 0.000 | 0.000 | 0.000 | 0.000 | 0.015 | 199 |
| E03 | 0.000 | 0.000 | 0.777 | 0.118 | 0.000 | 0.000 | 0.000 | 0.105 | 831 |
| E05 | 0.000 | 0.000 | 0.000 | 0.790 | 0.104 | 0.000 | 0.000 | 0.106 | 376 |
| E06 | 0.000 | 0.000 | 0.000 | 0.000 | 0.857 | 0.070 | 0.000 | 0.074 | 230 |
| E08 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.933 | 0.017 | 0.051 | 178 |
| E09 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.914 | 0.086 | 35 |

| FY 11-12 | E00-E01 | E02 | E03 | E05 | E06 | E08 | E09 | ATTRITES | Total |
|---|---|---|---|---|---|---|---|---|---|
| E00-E01 | 0.307 | 0.596 | 0.032 | 0.000 | 0.000 | 0.000 | 0.000 | 0.066 | 349 |
| E02 | 0.000 | 0.123 | 0.814 | 0.000 | 0.000 | 0.000 | 0.000 | 0.064 | 236 |
| E03 | 0.000 | 0.000 | 0.713 | 0.177 | 0.000 | 0.000 | 0.000 | 0.110 | 826 |
| E05 | 0.000 | 0.000 | 0.000 | 0.689 | 0.134 | 0.000 | 0.000 | 0.177 | 395 |
| E06 | 0.000 | 0.000 | 0.000 | 0.000 | 0.803 | 0.060 | 0.000 | 0.137 | 234 |
| E08 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.876 | 0.022 | 0.102 | 186 |
| E09 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.971 | 0.029 | 34 |

| FY 12-13 | E00-E01 | E02 | E03 | E05 | E06 | E08 | E09 | ATTRITES | Total |
|---|---|---|---|---|---|---|---|---|---|
| E00-E01 | 0.434 | 0.465 | 0.031 | 0.000 | 0.000 | 0.000 | 0.000 | 0.069 | 318 |
| E02 | 0.000 | 0.013 | 0.908 | 0.000 | 0.000 | 0.000 | 0.000 | 0.079 | 229 |
| E03 | 0.000 | 0.000 | 0.765 | 0.088 | 0.000 | 0.000 | 0.000 | 0.147 | 763 |
| E05 | 0.000 | 0.000 | 0.000 | 0.750 | 0.088 | 0.000 | 0.000 | 0.162 | 420 |
| E06 | 0.000 | 0.000 | 0.000 | 0.000 | 0.875 | 0.044 | 0.000 | 0.081 | 248 |
| E08 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.944 | 0.011 | 0.044 | 180 |
| E09 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 1.000 | 0.000 | 37 |

| FY 13-14 | E00-E01 | E02 | E03 | E05 | E06 | E08 | E09 | ATTRITES | Total |
|---|---|---|---|---|---|---|---|---|---|
| E00-E01 | 0.435 | 0.441 | 0.025 | 0.000 | 0.000 | 0.000 | 0.000 | 0.099 | 363 |
| E02 | 0.000 | 0.020 | 0.922 | 0.000 | 0.000 | 0.000 | 0.000 | 0.059 | 153 |
| E03 | 0.000 | 0.000 | 0.774 | 0.095 | 0.000 | 0.000 | 0.000 | 0.132 | 813 |
| E05 | 0.000 | 0.000 | 0.000 | 0.752 | 0.082 | 0.000 | 0.000 | 0.166 | 391 |
| E06 | 0.000 | 0.000 | 0.000 | 0.000 | 0.807 | 0.091 | 0.000 | 0.102 | 264 |
| E08 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.944 | 0.011 | 0.044 | 180 |
| E09 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.949 | 0.051 | 39 |

| FY 14-15 | E00-E01 | E02 | E03 | E05 | E06 | E08 | E09 | ATTRITES | Total |
|---|---|---|---|---|---|---|---|---|---|
| E00-E01 | 0.451 | 0.440 | 0.058 | 0.000 | 0.000 | 0.000 | 0.000 | 0.051 | 448 |
| E02 | 0.000 | 0.000 | 0.933 | 0.000 | 0.000 | 0.000 | 0.000 | 0.055 | 165 |
| E03 | 0.000 | 0.000 | 0.814 | 0.068 | 0.000 | 0.000 | 0.000 | 0.118 | 806 |
| E05 | 0.000 | 0.000 | 0.000 | 0.810 | 0.056 | 0.000 | 0.000 | 0.134 | 374 |
| E06 | 0.000 | 0.000 | 0.000 | 0.000 | 0.888 | 0.060 | 0.000 | 0.052 | 250 |
| E08 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.924 | 0.025 | 0.051 | 197 |
| E09 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.974 | 0.026 | 39 |

| FY15-16 | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | **E00-E01** | **E02** | **E03** | **E05** | **E06** | **E08** | **E09** | **ATTRITES** | **Total** |
| **E00-E01** | 0.400 | 0.477 | 0.053 | 0.000 | 0.000 | 0.000 | 0.000 | 0.070 | 413 |
| **E02** | 0.000 | 0.126 | 0.854 | 0.000 | 0.000 | 0.000 | 0.000 | 0.020 | 198 |
| **E03** | 0.000 | 0.000 | 0.778 | 0.103 | 0.000 | 0.000 | 0.000 | 0.119 | 837 |
| **E05** | 0.000 | 0.000 | 0.000 | 0.825 | 0.109 | 0.000 | 0.000 | 0.067 | 359 |
| **E06** | 0.000 | 0.000 | 0.000 | 0.000 | 0.889 | 0.074 | 0.000 | 0.037 | 243 |
| **E08** | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.969 | 0.000 | 0.031 | 196 |
| **E09** | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 1.000 | 0.000 | 42 |

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